RESEARCH ARTICLE

Endoscopic Analysis of Wave Propagation with Ag-nanoparticles in Curved Tube Having Permeable Walls

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Abstract: Objective: The purpose of this article is to analyze the effects of different shaped Ag-nanoparticles on peristaltic flow through curved tube with permeable walls.

Method: In the considered endoscope the inner tube is rigid while outer tube experience a sinusoidal wave and both tubes make an annulus. Bricks, cylinders and platelets are different shaped Ag-nanoparticles. Toroidal coordinate system is used to examine the nature of Ag-nanoparticles mathematically in the curved tube with viscous fluid. Analysis is performed with the consideration of low Reynolds number and long wavelength approximation. Perturbation approximation is used to solve the problem and achieve results for pressure gradient, pressure rise, axial velocity and stream functions.

Results: The effects of various parameters such as Grashoff's number, Darcy's number, radius of the endoscope and amplitude ratio on flow variables have been discussed graphically. The contemporary investigation has revealed that the temperature profile shows a decrease for larger shape factor of Ag-nanoparticles. Pressure gradient exhibits higher results with larger Darcy's number. Also trapped bolus tend to have bigger size for greater shape factor.

Conclusion: Temperature profile for the nanofluid decreases with the increase in shape factor m of nanoparticles. The trapping phenomena reveal that the size of inner bolus appears larger for platelet nanoparticles as compared to brick and cylinder nanoparticles.

Keywords: Curved annulus, Ag-nanoparticles, peristaltic pumping, porous boundary, perturbation solution, permeable wall.

1. INTRODUCTION

Transport phenomena of fluid occurring due to the progressive wave propagation along the elastic wall of channel are said to be peristalsis. In the recent years, peristaltic transport turns into a pre-eminent subject for the researchers as it is an essential transport component involved in numerous tubular human organs. The gastrointestinal tract, the urethra, the ureter, circulation of blood, swallowing and disintegration of food, movement of ovum and embryo in the tubes are few noteworthy examples, out of many, representing the phenomena of peristalsis. A vast application has been observed in mechanical application where direct contact with boundaries is denied as the principle of symmetric relaxation and contraction is followed by peristaltic wave. Sterile liquid transport, transport of destructive liquids, roller and finger pumps, heart lung machine and cell separation are some peristaltic applications. Due to extensive involvement and huge applications in industry, biomedical engineering and physiology, attention has been given to peristalsis for Newtonian and Non-Newtonian fluids under different situations. The inaugural in this direction was done by Latham [1] and Shapiro et al. [2]. Eventually, some more attempts were made and can be seen in the articles [3-10].

Endoscopy, since its introduction, has had a dramatic impact on the implementation of medical and industrial investigations. It opened a window of analytical potential outcomes and diagnostic possibilities that keeps on growing right up 'till the present time, impacting the way we oversee symptomatic patients and screen for disease. From dynamic aspect there is no difference between catheter and endoscope. Catheters can be adapted for gastrointestinal, urological, cardiovascular, neurovascular and ophthalmic applications by revising the material and refining the procedure through which catheters are manufactured. Access to surgical instruments, drainage, administration of fluid and gases and variety of different tasks are basic functions of catheters depending upon its type. Moreover, an embedded catheter will change the pressure distribution and flow field. To analyze the im-
impact of endoscope over peristalsis, various investigations are carried out such as [11-17].

Low thermal conductivity of ordinary fluids e.g., oil, water, ethylene glycol mixture has proven to be the main obstacle for improving the heat transfer processes in such fluids. As compared to the fluids, metals tend to have greater thermal conductivity. Use of high thermal conductivity of metals, to improve the thermal conductivity of fluids prompted the possibility of nanofluids. Preparation of such fluids involve suspension of nanometer sized particles in conventional fluid. Due to this suspension, the resulting fluid thus has higher effective thermal conductivity as compared to the base fluid. Choi [18] was the first one to introduce nanofluids. Metal oxides, metals and nitride/carbide are the most frequently used nanoparticles whereas water, ethylene and oils are the conventional fluids used as base fluid. Numerous industrial and engineering devices have observed effective functionality of nanofluids and thus nanoparticles are used in several appliances such as automobile and IT industry, coolants in nuclear reactors etc. An interesting utilization of nanoparticles in medicine is delivery of drug, light and heat to specific cells such as cancer cells. Nanoparticles are designed so that they are pulled to festered cells, which permit coordinate treatment to those cells. This procedure helps lessen the harm to healthy cells and takes into consideration prior discovery of disease. As nanofluid exhibits such huge advantages, miscellaneous investigations were done to analyze different characteristics of respective fluids [19-29].

The contribution of permeable medium is undeniable for biofluid and mechanical problems. Darcy’s law is used to get the fluid flow in porous medium while Navier Stokes is utilized for free flow region. In 1967 Beavers and Joseph considered permeable surface with couple flow motion. Blood vessels, bladders having stones, tumorous vessels, kidneys, bile duct, cardiovascular beds are some popular examples exhibiting permeability. Due to such crucial participation of permeable medium in human physiology, researchers have shown a keen interest in this aspect [30-33].

Keeping an eye on previous work, the purpose of this study is to assemble Ag-nanoparticles with peristalsis. Thus, we investigate different shape factor of Ag-nanoparticles in the presence of endoscope in a curved tube with permeable walls. Long wavelength and low Reynolds number assumptions are used to the dimensionless equations. A Perturbation analysis of curvature parameter is presented. Plots for different parameters are analyzed and displayed.

2. MATERIALS AND METHODS

2.1. Materials (Formulation of the Problem)

We are interested to investigate the peristaltic transport of incompressible, laminar and viscous nanofluid in the region between two curved annular tubes. A sinusoidal wave of speed $c$ travels along the walls of outer tube with wave amplitude $b$ and wavelength $\lambda$. Inner tube is considered to be rigid and has constant temperature $T_o$ while outer tube maintains temperature $T_1$. The mathematical formulation model for curved tube is, a rigid circular tube of radius $a_z$ wrapped in the form of a circle of radius $k$ and endoscope as coaxial tube with radius $a_1$. Due to the curved nature of tube, curvature parameter is also taken into account. Gold nanoparticles with shape factors brick, cylinder and platelet are considered along with blood.

Mathematically, two wall surfaces can be described as:

$$
\bar{R}_1 = a_1, \\
\bar{R}_2 = a_z + b\sin \left( \frac{2\pi}{\lambda} (Z - ct) \right),
$$

(1)

Fig. (1a). Geometry of problem.

Fig. (1b). Coordinate system.
The continuity equation for incompressible fluid in the toroidal coordinates is defined below [15-17]:

$$\frac{\partial \tilde{U}}{\partial R} + \frac{\partial \tilde{V}}{\partial \theta} + \frac{\partial \tilde{W}}{\partial z} + \frac{1}{k} \frac{\partial \tilde{W}}{\partial z} = 0. \quad (2)$$

The $\tilde{R}$, $\tilde{\theta}$ and $\tilde{Z}$ components of momentum equation using toroidal coordinate are given as [15-17]:

$$\rho_u \left( \frac{\partial \tilde{U}}{\partial t} + \tilde{U} \frac{\partial \tilde{U}}{\partial R} + \tilde{V} \frac{\partial \tilde{U}}{\partial \theta} + \tilde{W} \frac{\partial \tilde{U}}{\partial z} \right) + \frac{\partial P}{\partial R} + \frac{\partial}{\partial \theta} \left( \tilde{U} \frac{\partial \tilde{U}}{\partial \theta} \right) + \frac{\partial}{\partial z} \left( \tilde{U} \frac{\partial \tilde{U}}{\partial z} \right) \left( \frac{\partial \tilde{U}}{\partial z} + \frac{k}{k + R \cos \theta} \frac{\partial \tilde{W}}{\partial z} \right) = \mu \left( \frac{\partial^2 \tilde{U}}{\partial R^2} + \frac{1}{R} \frac{\partial \tilde{U}}{\partial \theta} + \frac{\partial^2 \tilde{U}}{\partial \theta^2} \right) \quad (3)$$

$$\rho_u \left( \frac{\partial \tilde{V}}{\partial t} + \tilde{U} \frac{\partial \tilde{V}}{\partial R} + \tilde{V} \frac{\partial \tilde{V}}{\partial \theta} + \tilde{W} \frac{\partial \tilde{V}}{\partial z} \right) + \frac{\partial P}{\partial \theta} + \frac{\partial}{\partial \theta} \left( \tilde{U} \frac{\partial \tilde{V}}{\partial \theta} \right) + \frac{\partial}{\partial z} \left( \tilde{U} \frac{\partial \tilde{V}}{\partial z} \right) \left( \frac{\partial \tilde{V}}{\partial z} + \frac{k}{k + R \cos \theta} \frac{\partial \tilde{W}}{\partial z} \right) = \mu \left( \frac{\partial^2 \tilde{V}}{\partial R^2} + \frac{1}{R} \frac{\partial \tilde{V}}{\partial \theta} + \frac{\partial^2 \tilde{V}}{\partial \theta^2} \right) \quad (4)$$

$$\rho_u \left( \frac{\partial \tilde{W}}{\partial t} + \tilde{U} \frac{\partial \tilde{W}}{\partial R} + \tilde{V} \frac{\partial \tilde{W}}{\partial \theta} + \tilde{W} \frac{\partial \tilde{W}}{\partial z} \right) + \frac{\partial P}{\partial z} + \frac{\partial}{\partial R} \left( \tilde{U} \frac{\partial \tilde{W}}{\partial R} \right) + \frac{\partial}{\partial \theta} \left( \tilde{U} \frac{\partial \tilde{W}}{\partial \theta} \right) + \frac{\partial}{\partial z} \left( \tilde{U} \frac{\partial \tilde{W}}{\partial z} \right) \left( \frac{\partial \tilde{W}}{\partial z} + \frac{k}{k + R \cos \theta} \frac{\partial \tilde{W}}{\partial z} \right) = \mu \left( \frac{\partial^2 \tilde{W}}{\partial R^2} + \frac{1}{R} \frac{\partial \tilde{W}}{\partial \theta} + \frac{\partial^2 \tilde{W}}{\partial \theta^2} \right) \quad (5)$$

Energy equation in the presence of heat generation for a nanofluid is given as [20],

$$\left( \frac{\rho C_p}{k} \right)_{nf} \left( \tilde{U} \frac{\partial \tilde{T}}{\partial R} + \tilde{V} \frac{\partial \tilde{T}}{\partial \theta} + \tilde{W} \frac{\partial \tilde{T}}{\partial z} \right) = k_{nf} \left( \frac{1}{R} \frac{\partial}{\partial R} \left( R \frac{\partial \tilde{T}}{\partial R} \right) + \frac{\partial^2 \tilde{T}}{\partial \theta^2} \right) + \frac{\partial}{\partial z} \left( \frac{\partial \tilde{T}}{\partial z} \right) \left( \frac{\partial \tilde{T}}{\partial z} + \frac{k}{k + R \cos \theta} \frac{\partial \tilde{T}}{\partial z} \right) \quad (6)$$

For the fixed frame, the boundary condition is given as:

$$\tilde{T} = T_0, \tilde{W} = 0, \text{ at } R = R_c = a_1, \quad \tilde{T} = T_1, \tilde{W} = \tilde{W}_b, \text{ at } R = R_c = b \sin \left[ \frac{2\pi}{\lambda} (Z - ct) \right] + a_2, \quad (7)$$

where, $T_1$ and $T_0$ is the temperature of outer and inner tube and $\tilde{W}_b$ is the slip velocity at $R_2$ as suggested in [30]:

$$\tau_{nf} = \beta_n (W_b - Q), \quad (8)$$

where, $Q$ is Darcy’s velocity given by:

$$Q = -\frac{k^*}{\mu} (\nabla \tilde{P} - (\rho \beta_n)_{nf} \nabla (\tilde{T} - T_1)) \quad (9)$$

where, $k^*$ is the permeability constant.

For the stated nanofluid model, viscosity $\mu_{nf}$, specific heat and density are defined as mentioned in [5]:

$$(\rho c_p)_{nf} = (\rho_c)_{nf} \phi (\theta - 1) (\rho_c)_{nf}, \quad \mu_{nf} = \frac{\mu}{1 + \frac{1}{3} n_c}, \quad K_{nf} = \frac{K_{nf}}{(\rho C_p)_{nf}} \quad (10)$$

The expression for thermal heat conductivity of nanofluids is expressed as:

$$\frac{K_{nf}}{K_f} = \frac{(n - 1) k_f + k - (n - 1)(k_f - k)}{k_f + (n - 1) k}, \quad (11)$$

Here, $n$ signify shape factor of nanoparticles given by $3/\psi_3$, where $\psi$ represents sphericity of the particle and is determined by the formation of nanoparticle. For cylindrical nanoparticle $n = 6$ or $\psi = 1/2$ while for spherical nanoparticle $\psi = 1$ or $n = 6$. Here, in this investigation we have taken $n = 6$ i.e., considered spherical shape.

The axial velocity component is the most effective component so take the axial velocity as $(0, 0, \tilde{W})$. The following transformation is used to swap from $(\tilde{R}, \tilde{Z}, T)$ fixed frame $(\tilde{r}, \tilde{\theta}, \tilde{z})$ to wave frame,

$$\tilde{z} = Z - c \tilde{t}, \tilde{P}(\tilde{r}, \tilde{\theta}, \tilde{z}) = P(Z, R, \tilde{T}), \quad \tilde{T} = \tilde{W} - c \tilde{t}, \quad (12)$$

in which $\tilde{u}$, $\tilde{W}$ and $\tilde{P}$ are the components of velocity and pressure in wave frame.

Bring out the following dimensionless quantities:

$$w = \frac{\tilde{w}}{c}, \quad \zeta = \frac{a_1}{k}, \quad u = \frac{\lambda}{a_c}, \quad R_f = \frac{a_c \rho f}{\mu f}, \quad \tau = \frac{\tilde{r}}{a_1}, \quad \varepsilon = \frac{\sin(2\pi \tilde{z}) + 1}{\lambda}, \quad (13)$$

$$r = \frac{\tilde{r}}{a_2}, \quad \tilde{\theta} = \theta, \quad \delta = \frac{a_2}{\lambda}, \quad \zeta = \frac{\tilde{z}}{\lambda}, \quad \gamma = \frac{a_2 M_0}{k_f (\tilde{T}_1 - \tilde{T}_0)}.$$
In Eq. (13), \( G_r \) is the Grashof number, \( p \) is the pressure, \( \tilde{\theta} \) is dimensionless temperature, \( R_e \) is Reynolds number, \( \gamma \) is dimensionless heat source parameter, \( \zeta \) is curvature parameter, darcy number and \( \delta \) represent wave number. After employing the lubrication approach, Eqs. (2-6) take the form:

\[
\frac{\partial p}{\partial r} = 0, \quad (14)
\]

\[
\frac{\partial p}{\partial \theta} = 0, \quad (15)
\]

\[
\frac{1}{\zeta \cos \theta + 1} \frac{\partial p}{\partial z} + \frac{1}{r} \frac{\partial}{\partial r} \left( \frac{1}{p} \frac{\partial p}{\partial r} \right) + \frac{1}{r} \frac{\partial}{\partial r} + \frac{\partial}{\partial \theta} \frac{\partial \theta}{\partial r} = 0. \quad (16)
\]

The boundary conditions in the wave frame are described as:

\[
\tilde{\theta} = 1, \quad w = -1, \quad \text{at} \quad r = r_1 = \varepsilon, \quad (18)
\]

\[
\tilde{\theta} = 0, \quad w = w_g = -1, \quad \text{at} \quad r = r_2 = \varepsilon \sin(2\pi z) + 1, \quad (18)
\]

Dimensionless volume flow rate is given as:

\[
q = F + \frac{1}{2} - \frac{\varepsilon^2}{2} + \frac{\varepsilon^2}{4} \quad (19)
\]

\[
F = \int_{r_1}^{r_2} r w dr \quad (20)
\]

3. METHODS

3.1. Solution of the Problem

In order to get the expression for velocity and temperature according to the given boundary condition, we consider the following:

\[
\tilde{\theta}(r, z, t) = \tilde{\theta}_0(r) + \zeta \cos(\theta) \tilde{\theta}_1(r) + \ldots \quad (21)
\]

\[
w(r, z, t) = w_0(r) + \zeta \cos(\theta) w_0(r) + \ldots \quad (21)
\]

Substituting Eq. (21) to Eqs. (16) to (18) and equating like powers of \( \zeta \cos(\theta) \), we obtain the following systems and their solutions

**Zero**\(^{th}\) order system and its solution

\[
\frac{K_f}{K_{nf}} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{\partial^2 \tilde{\theta}}{\partial r^2} = 0, \quad (22)
\]

\[
\frac{\partial^2 \tilde{\theta}_0}{\partial r^2} + \frac{1}{r} \frac{\partial \tilde{\theta}_0}{\partial r} + G_r (\rho \beta)_{nf} \frac{\mu_f}{\mu_{nf}} \tilde{\theta}_0 = \frac{\mu_f}{\mu_{nf}} \frac{dp}{dz}, \quad (23)
\]

\[
\tilde{\theta}_0(r_1) = 1, \quad w_0(r_1) = -1, \quad (24)
\]

\[
\tilde{\theta}_0(r_2) = 0, \quad w_0(r_2) = w_g - 1. \quad (25)
\]

The exact solution in this order is given as:

\[
\tilde{\theta}_0(r) = C_1 \ln r + C_2 - \frac{\gamma K_f}{4} r^2, \quad (26)
\]

\[
w_0(r) = \frac{\gamma K_f}{4} r^2 - G_r (\rho \beta)_{nf} \frac{\mu_f}{\mu_{nf}} \left[ \frac{\gamma K_f}{64} + C_1 \left( \frac{r^2 \ln r}{2} - \frac{r^2}{2} \right) + C_2 \right], \quad (27)
\]

**First order system and its solution**

\[
\frac{\partial^2 \tilde{\theta}_1}{\partial r^2} + \frac{1}{r} \frac{\partial \tilde{\theta}_1}{\partial r} + \tilde{\theta}_0 + \frac{\partial^2 \tilde{\theta}_0}{\partial r^2} + r \frac{K_f}{K_{nf}} = 0, \quad (28)
\]

\[
\frac{1}{r} \frac{\partial w_1}{\partial r} + \frac{1}{r^2} \frac{\partial^2 w_1}{\partial r^2} + 2 \frac{\partial \tilde{\theta}_0}{\partial r} + \frac{r}{K_{nf}} K_r (\rho \beta)_{nf} \frac{\mu_f}{\mu_{nf}} \tilde{\theta}_0 = 0, \quad (29)
\]

\[
\tilde{\theta}_1(r_1) = 0, \quad w_1(r_1) = 0, \quad (30)
\]

\[
\tilde{\theta}_1(r_2) = 0, \quad w_1(r_2) = w_g. \quad (31)
\]

Solution is obtained by substituting Eqs. (26, 27) into Eqs. (28, 29) and is given as follow:

\[
\tilde{\theta}_1(r) = r C_1 + C_4 \left( \frac{\gamma K_f r^2}{8} - C_1 \ln r \right) \frac{1}{r} \left( \frac{K_f}{K_{nf}} \right) + \frac{1}{r} \left( \frac{C_r r^2}{16} \right) \quad (32)
\]

\[
w_1(r) = C_1 + C_2 + \frac{r}{2} \left[ \frac{3 \gamma K_f}{42} + G_r (\rho \beta)_{nf} \frac{\mu_f}{\mu_{nf}} \left( \frac{\gamma K_f}{32} + \frac{C_r r^2}{2} + \frac{C_r r^2 \ln r}{4} + \frac{C_r r^2}{4} - \frac{C_1}{4} \right) \right]
\]

\[
\left( \frac{C_r r^2}{2} - C_1 \ln r - C_1 \ln r \right) \left( \frac{1}{2} - \frac{3 \gamma K_f}{42} + G_r (\rho \beta)_{nf} \frac{\mu_f}{\mu_{nf}} \left( \frac{\gamma K_f}{48} + \frac{C_r r^2}{4} \right) \right) \quad (33)
\]

where all the C's are constants and defined in appendix.

The pressure gradient is defined as:

\[
\frac{dp}{dz} = \frac{F - t_1}{t_2}, \quad (34)
\]

where \( t_1 \) and \( t_2 \) are calculated by Mathematica.
4. RESULTS

Fig. (2). (a, b), Axial velocity for Grashroff number $G_r$ with shape factor of bricks.

Fig. (3). (a, b), Axial velocity for Grashroff number $G_r$ with shape factor of cylinders.

Fig. (4). (a, b), Axial velocity for Grashroff number $G_r$ with shape factor of platelets.
Figs. (5). (a, b), Axial velocity for heat source (sink) $\gamma$ with shape factor of bricks.

Fig. (6). (a, b), Axial velocity for heat source (sink) $\gamma$ with shape factor of cylinders.

Fig. (7). (a, b), Axial velocity for heat source (sink) $\gamma$ with shape factor of platelets.
Fig. (8). (a, b), Axial velocity for amplitude ratio $\varphi$ with shape factor of bricks.

Fig. (9). (a, b), Axial velocity for amplitude ratio $\varphi$ with shape factor of cylinders.

Fig. (10). (a, b), Axial velocity for amplitude ratio $\varphi$ with shape factor of platelets.
Fig. (11). (a, b), Axial velocity for Darcy's number $d$ with shape factor of bricks.

Fig. (12). (a, b), Axial velocity for Darcy's number $d$ with shape factor of cylinders.

Fig. (13). (a, b), Axial velocity for Darcy's number $d$ with shape factor of platelets.
Fig. 14. (a, b, c), Pressure gradient for different values of Grashroff number $G_r$ for (a) Bricks (b) Cylinders (c) Platelets.

Fig. 15. (a, b, c), Pressure gradient for different values of heat source (sink) parameter $\gamma$ for (a) Bricks (b) Cylinders (c) Platelets.

Fig. 16. (a, b, c), Pressure gradient for different values of amplitude ratio $\phi$ for (a) Bricks (b) Cylinders (c) Platelets.

Fig. 17. (a, b, c), Pressure gradient for different values of Darcy's number $d$ for (a) Bricks (b) Cylinders (c) Platelets.
Fig. (18). (a, b, c), Pressure rise for different values of Grashoff number $G_r$ for (a) Bricks (b) Cylinders (c) Platelets.

Fig. (19). (a, b, c), Pressure rise for different values of heat source (sink) parameter $\gamma$ for (a) Bricks (b) Cylinders (c) Platelets.

Fig. (20). (a, b, c), Pressure rise for distinct values of amplitude ratio $\varphi$ for (a) Bricks (b) Cylinders (c) Platelets.

Fig. (21). Streamlines for Gold nanoparticle with shape factor of bricks for Grashoff's number (a) $G_r = 2$, (b) $G_r = 3$, (c) $G_r = 4$. 
Figs. (22). Streamlines for Gold nanoparticle with shape factor of cylinders for distinct values of (a) \( G_s = 2 \), (b) \( G_s = 3 \), (c) \( G_s = 4 \).

Figs. (23). Streamlines for Gold nanoparticle with shape factor of platelets for distinct values of (a) \( G_s = 2 \), (b) \( G_s = 3 \), (c) \( G_s = 4 \).

Fig. (24). Streamlines for Gold nanoparticle with shape factor of bricks for distinct values of (a) \( \gamma = 0.1 \), (b) \( \gamma = 0.5 \), (c) \( \gamma = 0.9 \).
Fig. (25). Streamlines for Gold nanoparticle with shape factor of cylinders for distinct values of (a) $\gamma = 0.1$, (b) $\gamma = 0.5$, (c) $\gamma = 0.9$.

Fig. (26). Streamlines for Gold nanoparticle with shape factor of platelets for distinct values of (a) $\gamma = 0.1$, (b) $\gamma = 0.5$, (c) $\gamma = 0.9$.

Fig. (27). Streamlines for Gold nanoparticle with shape factor of bricks for distinct values of (a) $\phi = 0.03$, (b) $\phi = 0.05$, (c) $\phi = 0.07$. 
Fig. (28). Streamlines for gold nanoparticle with shape factor of cylinders for distinct values of (a) $\phi = 0.03$, (b) $\phi = 0.05$, (c) $\phi = 0.07$.

Fig. (29). Streamlines for gold nanoparticle with shape factor of platelets for distinct values of (a) $\phi = 0.03$, (b) $\phi = 0.05$, (c) $\phi = 0.07$.

Fig. (30). Streamlines for gold nanoparticle with shape factor of bricks for distinct values of (a) $d = 0.008$, (b) $d = 0.009$, (c) $d = 0.01$. 
Fig. (31). Streamlines for gold nanoparticle with shape factor of cylinders for distinct values of (a) \(d = 0.008\), (b) \(d = 0.009\), (c) \(d = 0.01\).

Fig. (32). Streamlines for gold nanoparticle with shape factor of platelets for distinct values of (a) \(d = 0.008\), (b) \(d = 0.009\), (c) \(d = 0.01\).

Fig. (33). Streamlines for gold nanoparticle with shape factor of (a) Bricks \((m = 3.7)\), (b) Cylinders \((m = 4.9)\), (c) Platelets \((m = 5.7)\).
Table 1. Variation of temperature profile for different shape factor m with curvature $\zeta = 0.1$.

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Table 2. Variation of temperature profile for different shape factor m with curvature $\zeta = 0.5$.

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5. DISCUSSION

This section represents the discussion for the graphical results of velocity, pressure gradient, pressure rise and streamlines. These graphs are obtained by restricting the included parameters such as $G_r$, $\gamma$, $d$, $\theta$. Axial velocity $w$ demonstrates parabolic behavior against all the involved parameters in the region between tube having sinusoidal curve and endoscope. Figs. (2a, 2b) depict the impact of Grashoff's number on velocity profile while keeping other parameters fixed. Shape factor of bricks for Ag nanoparticles are considered and it is evident from the figure that axial velocity shows increasing attitude in the region $[0.1-0.66]$ for growing Grashoff's number but opposite trend is noticed in this region $[0.66-1]$. 3 dimensional axial velocity profile can be seen in Fig. (2b) for brick Ag nanoparticles for the variation of Grashoff's number. Figs. (3a, 3b) indicate how the velocity profile behaves for cylindrical Ag nanoparticles for varying Grashoff's number. It is viewed that velocity elevates in region $[0.1-0.66]$ and diminishes in region $[0.66-1]$. Fig. (3b) gives the 3 dimensional view of variation of velocity profile for cylindrical nanoparticles with modifying $G_r$. Similar trend is seen for both brick and cylindrical nanoparticles. Impact of platelet nanoparticles with different $G_r$ on velocity profile is captured in Figs. (4a, 4b) for both 2 and 3 dimensional flow. Variation is noticed to be alike as that of brick and cylindrical nanoparticles. Influence of heat source (sink) parameter $\gamma$ on axial velocity profile for shape factor of bricks for Ag nanoparti-
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The velocity profile shows a decrease in magnitude for increasing \( \gamma \). Figs. (5a, 5b) for 2D and 3D respectively. Velocity profile shows a decrease in magnitude for increasing \( \gamma \). Figs. (6a, 6b) are plotted to exhibit the trends followed by velocity profile for cylindrical Ag nanoparticles. Identical behaviour is observed for cylindrical and brick nanoparticles i.e., velocity profile shows a decline in its magnitude both for bricks and cylinders. Fig. (6b) portrays the 3D velocity for cylindrical Ag nanoparticles. Platelet nanoparticles with varying \( \gamma \) are plotted in Figs. (7a, 7b) for both 2 and 3 dimensions respectively. Bricks, cylinders and platelets are observed to give the same trend. Velocity is also influenced by amplitude ratio \( \phi \). Figs. (8a, 8b) describe the behaviour of velocity profile for different \( \phi \) with brick Ag nanoparticles. It is seen that velocity decreases in the region [0.1, 0.62] and increases in the region [0.62, 1] by increasing \( \phi \). An increase is noticed as we move from inner tube to the center of the region enclosed by the two tubes which is depicted in Figs. (9a, 9b) for cylindrical Ag nanoparticles giving both 2D and 3D velocity graphs with increasing amplitude ratio. Platelet nanoparticles show the same behaviour for amplitude ratio as that of cylindrical and brick nanoparticles and this behaviour is captured in Figs. (10a, 10b). Variation of Darcy’s number \( \Delta p \) affecting the velocity profile can be seen in Figs. (11a, 11b). Initially velocity increases for increasing \( \Delta p \) but as we move towards the porous tube, velocity decreases for elevating \( \Delta p \). In region [0.1, 0.74] velocity experiences a rise and then a decline is noticed in region [0.74, 1]. Figs. (11a, 11b) is 2 and 3-dimensional graph plotted to describe velocity profile for shape factor of brick Ag nanoparticles. Figs. (12a, 12b) portrays 2D and 3D view of axial velocity for cylindrical nanoparticles. Velocity boosts as we move from endoscope to central region i.e. 0.1 < \( r < 0.74 \) and a decline is noticed in the region 0.74 < \( r < 1 \). Figs. (13a, 13b) give the effects of platelet Ag nanoparticles for different values of darcy's number and they behave likely as bricks and cylinders.

Figs. (14-17) are sketched to describe the change in pressure gradient for different \( G_r \), \( \gamma \), \( \phi \) and \( \Delta p \). Figs. (14a-14c) depicts the behaviour of pressure gradient affected by Grashoff's number \( G_r \). It has been observed that the growth of buoyancy forces results in the decrease of pressure gradient. Shape factor of bricks, cylinders and platelets for Ag nanoparticle all give the same behaviour. Figs. (15a-15c) describes the behaviour of \( \Delta p / \Delta z \) against \( z \) for variational heat source (sink) parameter \( \gamma \). The amplitude of pressure is noticed to decrease as \( \gamma \) gets higher values with shape factors of bricks, cylinders and platelets. The variational change in pressure gradient due to amplitude ratio \( \phi \) with bricks, cylinders and platelets is shown in Figs. (16a-16c) respectively. Pressure gradient is noticed to decrease in the region \([-1, -0.5]\) and \([0, 0.5]\) and increase is observed in the region \([-0.5, 0]\) and \([0.5, 1]\) for elevating amplitude ratio \( \phi \). Figs. (17a-17c) is plotted to give the influence of darcy's number \( d \) on \( dp / dz \) by taking bricks, cylinders and platelets. An increase in darcy number \( d \) give rise to amplitude of pressure gradient.

To explain the pumping properties, it is crucial to know pressure rise per wavelength. Thus, Figs. (18-20) are plotted to depict the pressure rise for varying different parameters such as Grashoff's number \( G_r \), heat source parameter \( \gamma \), amplitude ratio and darcy's number \( d \). With expansion in flow rate, on common observation from these figures, pressure rise per wavelength decreases. Figs. (19a-19c) is used to analyze the behaviour of pressure rise for different values of \( G_r \). It is observed that pressure rise experiences a decline in elevating \( G_r \) through annulus. This trend is followed in both retrograde pumping region \( (q < 0, \Delta p > 0) \) and augmented pumping region \( (q > 0, \Delta p < 0) \). Also shape factor of bricks, cylinders and platelet Ag nanoparticle behave likely i.e., pressure rise drops for all these nanoparticles with increase in \( G_r \). Figs. (20a-20c) is plotted to the effects of heat source (sink) parameter \( \gamma \) on \( \Delta p \). It can be seen from fig that \( \Delta p \) decreases in retrograde pumping region as well as in augmented pumping region when \( \gamma \) has been increased. The impact of brick, cylinder and platelet nanoparticles is similar for \( \Delta p \) for different \( \gamma \). Figs. (21a-21c) give the effects of amplitude ratio \( \phi \) over \( \Delta p \). As the value of parameter increases, \( \Delta p \) decreases in retrograde and augmented pumping region. This behaviour is seen for brick, cylinder and platelet Ag nanoparticles.

An engrossing phenomena of trapping for peristaltic flow with an endoscope is described in Figs. (21-33). Ag nanoparticles with different shape factors are taken into consideration for this discussion. Pattern of flow in the region enclosed by catheter and curved tube is studied by plotting streamlines. Random behaviour of enclosed bolus is seen for variation of \( G_r \) along with closed streamlines and is portrayed in Fig. (21). From Fig it is seen that the number of trapped bolus decreases when \( G_r \) shifts from 2 to 3 and size is also observed to decrease as \( G_r \) further changes from 3 to 4. From Figs. (21-23), it is evident that the change in bolus appearance is concordant for all different shape factors considered. Effects of \( \gamma \) over trapping phenomena is studied in Fig. (24). It is witnessed that as \( \gamma \) changes from 0.1 to 0.5, number of trapped bolus decreases and the size also recedes as \( \gamma \) jumps to 0.9 from 0.5. Different shape factors are considered to give the harmonious behaviour for variation of \( \gamma \) and this argument is supported by Figs. (24-26). Fig. (27) is used to show the impact of \( \phi \) over trapping phenomena. As \( \phi \) is increased from 0.03 to 0.05, number of bolus increases and on further increasing \( \phi \) from 0.05 to 0.07 a decrease in number of bolus is seen. Brick, cylinder and platelet Ag Nanoparticles give the same trend which can be
verified with the help of Figs. (27-29). Decrease in size of bolus is more noticeable in cylinder and platelet particles as compared to brick nanoparticles. Variation of darcy's number is studied in Figs. (30-32). Initially, number of trapped bolus increases for increasing Darcy’s number then their size also increases with similar trend of darcy’s number. Visual study has revealed that all the considered shape factors of nanoparticles have shown likely behaviour. Fig. (33) is used to compare the effects of different shape factors of nanoparticles used. Temperature profile for curved channel with permeable walls having shape factor $m$ is presented in Table 1. It is witnessed that for greater value of shape factor $i.e.,$ for larger $m$ the temperature of the base fluid decreases. Also, it is interesting to see that the variation in curvature effects the temperature profile. With larger curvature parameter $\zeta$ low temperature is noticed as can be seen in Table 2.

**CONCLUSION**

A detailed mathematical analysis has been performed to observe the impact of Ag nanoparticles on the peristaltic flow through a curved tube with an endoscope inserted in it. Some observations of the present study made on the basis of graphical results are highlighted below

- Temperature profile of the nanofluid decreases with the increase in shape factor $m$ of nanoparticles.
- With an increment in curvature parameter, temperature of nanofluid recedes.
- Pressure gradient exhibits higher results with larger Darcy’s number.

**APPENDIX**

$$C_1 = -\frac{4\gamma}{K_f} + \frac{4\gamma}{K_n} - \frac{4\gamma}{r_1^2 r_2^2 - \gamma K_n f^2}, \quad C_2 = \frac{4\gamma}{r_1^2 r_2^2 - \gamma K_n f^2},$$

$$C_3 = \frac{1}{16(r_1^2 - r_2^2)}(-4C_1 r_1^2 + 8C_1 r_1^2 \ln r_1 - \gamma K_f r_1^4 + 4C_1 r_1^2 + 8C_1 r_1^2 \ln r_2 - \gamma K_f r_1^4),$$

$$C_4 = \frac{1}{2}(8C_1 r_1^2 \ln r_1 - 8C_1 r_1^2 \ln r_2 - \gamma K_f r_1^4 + \gamma K_f r_1^2),$$

$$C_5 = \frac{a_4}{a_6} + \frac{1}{\zeta \cos \theta}(\frac{\alpha}{\sqrt{\alpha}} a_4 - a_5), \quad C_6 = -a_3 - \frac{\ln r_1}{a_6} (a_4 + (\frac{1}{\zeta \cos \theta} a_4 - a_5),$$

$$C_7 = \frac{a_{11}}{a_{12}}, \quad C_8 = -r_1 (a_7 + r_1 \frac{a_{11}}{a_{12}}), \quad b_1 = -S \left( \frac{C_2 r_1}{4} - \frac{B r_3}{16} + C_1 (-\frac{r_2^2}{4} + r_1^2 \ln r_2) \right), \quad b_2 = -S \left( \frac{C_2 r_1}{4} - \frac{B r_3}{16} + C_1 (-\frac{r_2^2}{4} + r_1^2 \ln r_2) \right),$$

$$b_7 = S \left( -\frac{B r_3}{32} + C_1 (-\frac{r_2^2}{4} + r_1^2 \ln r_2) + \frac{C_2 r_1}{4} - \frac{B r_3}{16} - C_4 \ln r_1 \right),$$

- Axial velocity elevates as we move from endoscope to the center of annular region.
- For greater Grashoff's number $G_r$ and Darcy’s number $d$, axial velocity is larger.
- The inner bolus grows larger with increasing Darcy’s number.
- The trapping phenomena reveal that the size of inner bolus appears larger for platelet nanoparticles as compared to brick and cylinder nanoparticles.

**ETHICS APPROVAL AND CONSENT TO PARTICIPATE**

Not applicable

**HUMAN AND ANIMAL RIGHTS**

No Animals/Humans were used for studies that are base of this research.

**CONSENT FOR PUBLICATION**

Not applicable.

**CONFLICT OF INTEREST**

The authors declare no conflict of interest, financial or otherwise.

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\[ b_8 = S \left( -\frac{Br_3^4}{48} + C_1 \left( -\frac{3r_3^4}{16} + \frac{r_4^4}{4} \right) + C_2 \frac{r_4^4}{8} - \frac{C_3 r_4^2}{4} - \frac{C_4}{2} \right) , \]
\[ b_9 = S \left( -\frac{Br_2^4}{32} + C_1 \left( -\frac{r_2^4}{4} + \frac{r_4^2}{2} \right) + \frac{C_2 r_2^4}{4} - \frac{C_3 r_2^2}{2} - C_4 \ln r_2 \right) , \]
\[ b_{11} = S \left( -\frac{5Br_2^4}{32} + \frac{C_1}{2} \left( -r_2^2 + 3r_2^2 \ln r_2 \right) \right) \]
\[ b_{13} = 1 + \frac{\zeta}{1+\zeta} \cos \theta , \]
\[ a_1 = b_1 + \frac{Lr_2}{2A}, \quad a_2 = b_2 + \frac{Lr_2}{4A}, \quad a_3 = b_3 + \frac{Lr_2}{4A}, \quad a_4 = a_2 - a_3, \quad a_5 = a_1 - 1 - \frac{AdL}{b_{13}} , \]
\[ a_6 = \left( \ln r_1 - \ln r_2 \right) \left( 1 + \frac{\zeta r_2 \cos \theta}{\zeta \cos \theta} \right) \left( \frac{1+\zeta r_2 \cos \theta}{\zeta \cos \theta} \right) , \]
\[ a_9 = \frac{1}{2} \left( -\frac{9r_4^2}{4A} + b_9 - \frac{C_5}{r_2} \right) - \frac{1}{2} \left( -\frac{9r_4^2}{4A} + b_9 - \frac{C_5}{r_2} \right) , \]
\[ a_{10} = -1 - \frac{AdL}{b_{13}}, \quad a_{11} = -a_8 + \frac{r_5}{r_2} a_7 , \]
\[ a_{12} = r_2 \frac{r_2}{r_2 - \frac{\sqrt{\alpha}}{\alpha} + \frac{\sqrt{\alpha} r_2 \zeta \cos \theta}{\alpha} - \frac{\sqrt{\alpha} r_2 \zeta \cos \theta}{\alpha} \ln r_2} {1+\zeta r_2 \cos \theta} , \]
\[ + \frac{\sqrt{\alpha} r_2 \zeta \cos \theta}{\alpha} - \frac{\sqrt{\alpha} r_2 \zeta \cos \theta}{\alpha} \frac{1}{1+\zeta r_2 \cos \theta} \]

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