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Abstract: Background: The permanent magnet synchronous linear motor is a strongly coupled, nonlinear system. It has been applied in many fields, especially in the field of machining lathes and rail transportation. In order to ensure the permanent magnet synchronous linear motor has good dynamic performance and robustness, sliding mode control is gradually applied to the control system of permanent magnet synchronous linear motor. However, in the traditional sliding mode control, the convergence speed is slow, and the robust performance is poor when the sliding surface is not reached.

Objective: The main purpose of this paper is to improve the dynamic performance and robustness of the permanent magnet synchronous linear motor during the process of approaching the sliding surface.

Methods: Firstly, the type of nonlinear curve with "small error reduction, large error saturation" is introduced to design a nonlinear integral speed controller with global robustness. Secondly, the gain rate time-varying reaching law is introduced to reduce "chattering". Finally, using a symbolic tangent function instead of a sign function in designing a sliding mode observer reduces fluctuations in load observations.

Results: Finally, the correctness and effectiveness of the control method are proved by simulation.

Conclusion: The results of the simulation show that the nonlinear integral sliding mode controller based on gain time-varying reaching law is shown to have good global robustness and dynamic performance.

Keywords: PMLSM, sliding mode control, global robustness, load observer, speed controller, time-varying reaching law.

1. INTRODUCTION

Compared with the traditional rotating electric machine, the Permanent Magnet Linear Synchronous Motor (PMLSM) can directly convert electrical energy into mechanical energy of linear motion and overcome the energy loss caused by rotating the rotating motion of the rotating electrical machine into linear motion through various transmission structures. The advantages of PMLSM are high precision, high speed, and "zero transmission". Because of these advantages, linear motors are widely used in CNC machine tools, rail transit, and other fields. With the continuous development of magnetic materials, materials such as NdFeB with high magnetic properties, strong coercivity, and high comprehensive have emerged and applied to various fields such as communication, transportation, and medical treatment. Therefore, PMLSM with NdFeB as the main magnetic material has also been favored in various fields.

The accuracy in the machining of CNC machine tools is generally high. After 2004, the precision boundary between precision machine tools and ultra-precision machine tools is 1 nm [1]. But permanent magnet linear synchronous motors also produce end effects, which are thrust fluctuations and normal force fluctuations, because of the unclosed magnetic circuit. Sliding mode control can design the switching plane of the system according to the dynamic characteristics expected by the system. The sliding mode controller moves the system state from the outside of the plane to the switching plane and arrives at the system origin along the switching plane. Therefore, it is very robust to interference and for modeling dynamics. However, the moving point is difficult to move along the sliding surface, but it traverses back and forth on both sides of the sliding surface; this problem is called "chattering". Therefore, domestic scholars combined some other excellent control methods with sliding mode control [2, 3]. Xiaoguang Zhang et al. [4] optimized the index reaching law and Weiye Liu et al. [5] optimized the switching function reaching law. These reaching laws improved the response speed under the premise of effectively reducing chattering. Lei Huang et al. [6] introduced sliding mode con-
control into the speed observer, which realizes the position sensorless control of the permanent magnet synchronous motor, which reduces the observation error and improves the accuracy. Peng Li [7], Bitao Zhang [8], Jun Liu [9], Liwei Zhang [10] et al. introduced integral sliding mode control into the motor control system, avoiding the noise introduced by differential action and improving the stability of the system. Meanwhile, sliding mode control is sensitive to parameter changes when reaching phase and the robust performance is poor. Chang and Hurmuzlu designed time-varying surfaces, which eliminate the reaching phase and obtain system insensitivity with respect to external disturbance and modeling uncertainty from the very beginning of the control process [11, 12]. Meanwhile, Yilmaz, Choi, Jayasuriya, and Eksin also improved and perfected the time-varying sliding surface [13-15]. Eliminating the reaching phase can also be obtained using Integral Sliding Mode Control (ISMC) proposed by Utkin and Shi [16]. In recent years, some researchers achieved some results by using ISMC in the converter, robot, and the power system [17-20].

In order to improve the response speed when using the ISMC to control linear motor, this paper designs nonlinear functions with small error reduction and large error saturation which is used in an integral sliding mode controller. In order to weaken the ‘chattering’ problem, this paper improved the traditional reaching law and introduced the gain time-varying reaching law. Therefore, the global robustness of the permanent magnet linear synchronous motor control system improved. At the same time, due to the sign function in the sliding mode observation of the load, the system switches have a high frequency, which causes fluctuations in the load observation value. In this paper, a symbolic tangent function is designed to replace the symbolic function, which can effectively solve the problem of fluctuations in the observation value caused by high-frequency switching of the system, thus improving the global robustness and dynamic performance of the control system.

2. MATERIALS AND METHODS

2.1. Mathematical Model of PMLSM

The principle of the permanent magnet linear synchronous motor and the permanent magnet synchronous rotating motor is basically the same because the permanent magnet linear synchronous motor can be regarded as the permanent magnet synchronous rotating motor which is axially deployed. Since the permanent magnet linear synchronous motor is a strongly coupled and nonlinear system, it is difficult to build a mathematical model of PMLSM. In order to obtain a simplified motor model, the following assumptions are made during the derivation:

(1) Ignore the influence of core saturation and temperature, assuming that the magnetic circuit is linear;

(2) The three-phase coil windings are identical, and the phase windings are evenly distributed and the parameters are the same, and the three phases are mutually different by 120 degrees;

(3) Excluding eddy current and hysteresis loss;

(4) The air gap between the primary and secondary is uniform;

(5) Ignoring the damping effect of the permanent magnet;

(6) The back electromotive force is sinusoidal;

The voltage equation of PMLSM in the synchronous rotating coordinate system (d-q coordinate system) is Eq. (1):

\[
\begin{align*}
\dot{u}_d &= R_i i_d + \omega_\psi q + \omega i_d q \\
\dot{u}_q &= R_i i_q + \omega_\psi d - \omega i_q d \\
\end{align*}
\]

(1)

The flux linkage equation is Eq. (2):

\[
\begin{align*}
\psi_d &= L_d i_d + \psi_f \\
\psi_q &= L_q i_q \\
\end{align*}
\]

Where: \(u_d, u_q\) are the components of the three-phase winding voltage on the d-q axis; \(i_d, i_q\) are the components of the three-phase winding current on the d-q axis; \(L_d, L_q\) are the components of the inductance on the d-q axis; \(\psi_f, \psi_q\) are the component of the windings flux linkage on the d-q axis; \(\psi_f\) is the permanent magnet fundamental excitation flux; \(R_s\) is the resistance of each phase winding; \(p\) is the differential operator; \(\omega\) is the angular velocity.

The electromagnetic thrust of the permanent magnet linear synchronous motor is Eq. (3):

\[
F_{cm} = p_n \frac{3\pi}{2\tau} \left[ \psi_f i_q + (L_d - L_q)i_d i_q \right]
\]

(3)

Where: \(F_{cm}\) is the electromagnetic thrust of the motor; \(p_n\) is the pole pair of the motor; \(\tau\) is the pole pitch of the motor.

In this paper, a surface-mounted permanent magnet linear synchronous motor is taken as an example, where \(L_d\) is equal to \(L_q\), so the electromagnetic torque equation can be simplified as Eq. (4):

\[
F_{cm} = p_n \frac{3\pi}{2\tau} \psi_f i_q
\]

(4)

The mechanical equation of a permanent magnet linear synchronous motor is Eq. (5):

\[
M \frac{dv}{dt} = F_{cm} - F_l - Bv
\]

(5)

Where: \(M\) is the mass of the primary motion part, \(v\) is the primary motion speed; \(F_l\) is the load of the motor; \(B\) is the viscous friction coefficient.

3. DESIGN OF NONLINEAR INTEGRAL SLIDING MODE SPEED CONTROLLER

3.1. Introduction of Symbolic Tangent Function

Liwei Zhang et al. [10] illustrated the feasibility of the application of nonlinear functions with the properties of amplifying small errors and saturating large errors. Therefore, this paper describes the feasibility and effectiveness of another nonlinear function with the properties of reducing small errors and saturating large errors in the whole-range
nonlinear controller. The symbolic tangent function with the above properties is introduced as follows Eq. (6):

\[
y(e) = \begin{cases} 
\alpha \sgn(e) & |e| \geq \alpha \\
\alpha \tan\left(\frac{\pi}{4\alpha} e\right) & |e| < \alpha 
\end{cases}
\]  

(6)

Where: \(\alpha\) is a constant, \(e\) is the difference between primary speed reference and actual speed value, take \(\alpha=5\) and draw a curve (calling the curve a symbolic tangent function) shown as Fig. (1). It can be seen from the figure that when \(e\) is between 0 and \(\alpha\), the value of \(y(e)\) is always less than \(e\) and the value is \(\tan(\pi e/4\alpha)\); when \(e\) is between \(-\alpha\) and 0, the value of \(y(e)\) is always greater than \(e\), the value is \(\tan(\pi e/4\alpha)\), the function has the function of reducing small errors. When \(e\) is greater than \(\alpha\), the error equals \(\alpha\), the function has the function of saturating large errors.

\[\text{Fig. (1). Symbolic tangent function. (A higher resolution / colour version of this figure is available in the electronic copy of the article).}\]

3.2. Design of Nonlinear Integral Sliding Mode Controller

In global sliding mode control, the global sliding mode function is usually constructed as follows Eq. (7):

\[
s = \dot{e} + ce - f(t)
\]

(7)

Where: \(c>0\), \(f(t)\) is a self-designed function. Generally, in order to achieve global sliding mode control, \(f(t)\) should satisfy the following properties:

1. \(f(0) = \dot{e}(0) + ce(0)\),
2. when \(t \rightarrow \infty\), \(f(t) \rightarrow 0\);
3. \(f(t)\) has the first derivative;

During the movement, the speed of the primary cannot be abrupt due to inertia, so the derivative of the velocity error is Eq. (8):

\[
f(t) = e(0)e^{-kt}
\]

(8)

Where: \(e(0)\) is the initial velocity error; \(k\) is a constant.

The global nonlinear integral sliding surface function is designed as follows:

\[
s = e + k_1 \int_0^t y(e)d\tau - e(0)e^{-kt}
\]

(9)

The derivative of Eq. (9) can be obtained as follows:

\[
\dot{s} = \dot{e} + k_2 y(e) + e(0)ke^{-kt}
\]

(10)

Order \(\dot{e} = -dv/dt\) and substitute Eq. (5) into Eq. (10):

\[
\dot{s} = -\frac{1}{M} (F_{cm} - F_T - Bv) + k_3 y(e) + e(0)ke^{-kt}
\]

(11)

In order to ensure that the system can quickly return the sliding surface, improve performance, introducing an exponential reaching law Eq. (12):

\[
\dot{s} = -\varepsilon \sgn(s) - \eta s
\]

(12)

Where: \(\varepsilon\) is the constant velocity approaching rate coefficient which is a constant greater than 0; \(\eta\) is the coefficient of exponential reaching law which is a constant greater than 0; \(k_1\) is a constant greater than 0.

The exponential reaching law makes the sliding mode appear as a band shape, which leads to the fact that it cannot approach to the zero point at last. The chattering occurs near the zero, which excites the high-frequency unmodeled characteristics. However, with the introduction of the constant velocity reaching law, the large value of \(\varepsilon\) will increase the chattering of the approaching process, causing the contradiction between the approaching speed and the chattering [5]. Therefore, a new type of reaching law with a time-varying gain is selected in this paper, and its expression is as follows:

\[
\dot{s} = -\frac{|s|}{|s|+\sigma} k_1 \sgn(s) - k_2 s
\]

(13)

The expression guarantees that when the sliding mode function \(s\) is continuously decreasing, \((|s|+k_1)/(|s|+\sigma)\) tends to be close to 0, so that the sliding mode motion can finally converge at the origin and also guarantees that the time-varying gain coefficient \((|s|+k_1)/(|s|+\sigma)\) is always less than the original coefficient \(\varepsilon\), which achieves the purpose of weakening chattering. In order to further weaken the chattering problem caused by the high-frequency switching of the symbolic function, this paper selects the \(\text{con}(s)\) function with relay characteristics instead of the symbolic function in Eq. (13), which is the new reaching law of gain time-varying used in this paper:

\[
\dot{s} = -\frac{|s|}{|s|+\sigma} k_1 \text{con}(s) - k_2 s
\]

(14)

Where: \(\text{con}(s) = s/(|s|+\Delta)\), \(\sigma, \Delta\) are both small positive numbers, \(k_2, k_3\) are both positive constant.

Combine Eqs. (4), (11), and (14) to obtain the controller formula:
Design of Sliding Mode Load Observer

Taking the cross-axis current $i_q$ and the primary motor velocity $v$ as observation objects, the sliding surface function $s$ is selected as Eq. (16):

$$s = \dot{v} - v$$  \hfill (16)$$

In the permanent magnet synchronous motor control system, it can be considered that the load $F_l$ of the motor does not mutate in a very short period of time, therefore, the derivative of $F_l$ is zero. Combining Eqs. (4, 5), the following equations are available:

$$\begin{cases}
\dot{v} = \frac{3 \pi p \psi_f}{B} i_q - \frac{B}{M} v - \frac{1}{M} F_l \\
\dot{F}_l = 0
\end{cases}$$  \hfill (17)$$

Therefore, the formula of the sliding mode observer is as follows:

$$\begin{cases}
\dot{\hat{v}} = \frac{3 \pi p \psi_f}{B} i_q - \frac{B}{M} \hat{v} - \frac{1}{M} \hat{F}_l + k_4 \text{sgn}(s) \\
\dot{\hat{F}_l} = k_4 \text{sgn}(s)
\end{cases}$$  \hfill (18)$$

Considering the fluctuation of the observed load due to high-frequency switching of $\text{sgn}(s)$, which affects the effect of the nonlinear integral sliding mode controller, the symbolic tangent function $\hat{y}(s)$ is used instead of the symbolic function. The Eq. (18) is rewritten as:

$$\begin{cases}
\dot{\hat{v}} = \frac{3 \pi p \psi_f}{B} i_q - \frac{B}{M} \hat{v} - \frac{1}{M} \hat{F}_l + k_4 \hat{y}(s) \\
\dot{\hat{F}_l} = k_4 \hat{y}(s)
\end{cases}$$  \hfill (19)$$

Define the primary speed error and the motor’s load error as $\tilde{v} = \hat{v} - v$, $\tilde{F}_l = \hat{F}_l - F_l$. The equations of error for the primary speed and motor load are obtained from Eqs. (17) and (19):

$$\begin{cases}
\dot{\tilde{v}} = \frac{B}{M} \hat{v} - \frac{1}{M} \hat{F}_l + k_4 \hat{y}(s) \\
\dot{\tilde{F}_l} = k_4 \hat{y}(s)
\end{cases}$$  \hfill (20)$$

Proof of Stability

Proof of the Stability of the Approach Process

Define the Lyapunov function as follows:

$$V = \frac{1}{2} s^2$$  \hfill (21)$$

Then,

$$\dot{V} = s \cdot \dot{s} = s(-\frac{B}{M} s - \frac{1}{M} \hat{F}_l + k_4 \hat{y}(s))$$  \hfill (22)$$

Therefore, Eq. (22) is always established, when $s > 0$, Eq. (25) therefore Eq. (26):

$$k_4 \leq \frac{\hat{F}_l}{M \hat{y}(s)}$$  \hfill (25)$$

If $s < 0$, then $\dot{y}(s) < 0$, therefore:

$$k_4 \leq \frac{\hat{F}_l}{M \hat{y}(s)}$$  \hfill (26)$$

When the system runs to steady-state, the primary velocity error is zero and the derivative of the velocity error is also zero Eq. (28):

$$\begin{cases}
\hat{v} = -\frac{B}{M} \hat{v} - \frac{1}{M} \hat{F}_l + k_4 \hat{y}(s) = 0 \\
\dot{\hat{F}_l} = k_4 \hat{y}(s)
\end{cases}$$  \hfill (28)$$

According to the stability theorem of Lyapunov function system, this system is stable in the approach process.
Simplify the formula:
\[ \dot{F_i} - \frac{k_5}{Mk_4} F_i = 0 \] (29)

According to the Rolls criterion, in order to ensure the stability of the system, it is necessary to ensure that the coefficients of the characteristic equation of the equation are always greater than zero.

\[ k_5 > 0 \] (30)

In summary, when \( k_4 \), \( k_5 \) is selected according to the above requirements, the stability of the load observer can be guaranteed.

Analyze the asymptotical stability of the sliding mode observer according to LaSalle’s Invariant Principle. Firstly, define a set \( \Omega \), which is non-empty and bounded. The solutions of the system are all in the set \( \Omega \) in the condition of \( t \geq 0 \). Suppose that \( \Gamma \) is the set of \( \dot{V} = 0 \) in the set \( \Omega \). N is the largest invariant set in the set \( \Gamma \). There are two cases for the solutions of the equation according to Eq. (23) when \( \dot{V} = 0 \). The first case is \( s = 0 \). The second case is \( s \neq 0 \), therefore \( \dot{s} = 0 \), so:

\[ \frac{B}{M} s - \frac{1}{M} \dot{F_i} + k_4 y(s) = 0 \] (31)

Both ends of the equation are multiplied by \( s \).

\[ (k_4 y(s) - \frac{1}{M} \dot{F_i}) s = \frac{B}{M} s^2 \geq 0 \] (32)

It can be seen that this formula contradicts Equation (24). Therefore, the second case is not right. So, the maximum invariant set N does not contain other solutions except \( s = 0 \). Therefore, the sliding mode observer is asymptotical stability.

4. RESULTS AND DISCUSSION

4.1. Algorithm Simulation, Result Analysis, and Discussion

In order to verify the control effect of the global nonlinear integral sliding mode controller based on gain time-varying reaching law designed in this paper, the Simulink module in MATLAB is used to construct the simulation model of the permanent magnet linear synchronous motor and its control system. The motor parameters used in this simulation are shown in Table 1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Primary quality M</td>
<td>50kg</td>
</tr>
<tr>
<td>Viscous friction coefficient B</td>
<td>0.1N·m·s</td>
</tr>
<tr>
<td>Pole distance ( \tau )</td>
<td>0.039m</td>
</tr>
<tr>
<td>Resistance of each phase winding ( R_s )</td>
<td>1Ω</td>
</tr>
<tr>
<td>Straight axis inductance ( L_d )</td>
<td>13.91mH</td>
</tr>
<tr>
<td>Cross-axis inductance ( L_q )</td>
<td>13.91mH</td>
</tr>
<tr>
<td>Number of pole pairs ( p_n )</td>
<td>3</td>
</tr>
<tr>
<td>Permanent magnet flux linkage ( \Psi_f )</td>
<td>0.2324Wb</td>
</tr>
</tbody>
</table>

The schematic diagram of the control system of PMLSM global robustness based on gain time-varying reaching law is shown in Fig. (2). As can be seen from the figure, the paper mainly uses a control method, which is a field-oriented vector with \( i_d \neq 0 \). The speed controller replaces the PI control in the traditional control method with the global nonlinear integral sliding mode controller designed in the paper. Due to the unknown quantity \( F_i \) in the speed controller designed in this paper, a sliding mode observer is introduced to observe the load.

Establishing a simulation model according to the system block diagram. In the simulation model, DC bus voltage \( u_d = 311V \). Considering the inverter as an ideal device, ignoring factors such as dead time and device turn-on voltage drop. The maximum simulation step size is \( 10^{-5}s \). In order to illustrate that the speed controller designed in the paper has good global robustness, the primary initial speed is set at...
0.3 m/s. Permanent magnet linear synchronous motor loads 100N at 0~15s and 150N at 0.15~0.2s. By comparing the changes of sliding mode surface s and velocity v between the traditional sliding mode speed controller and the nonlinear integral sliding mode controller designed in the paper when the permanent magnet linear synchronous motor starts up and suddenly increases load, it is further shown that the speed controller designed in the paper has the characteristics of global robustness.

Through simulation analysis, the primary speed curve of the linear motor can be obtained as shown in Fig. (3), where Fig. (4) is a partial enlarged view of the speed curve.

![Fig. (3). Permanent magnet linear synchronous motor primary speed curve. (A higher resolution / colour version of this figure is available in the electronic copy of the article).](image)

![Fig. (4). Local details of Permanent magnet linear synchronous motor primary speed curve. (A higher resolution / colour version of this figure is available in the electronic copy of the article).](image)

On the other hand, the curve of the nonlinear integral sliding mode surface s as shown in Fig. (5) can be obtained by simulation, and the detailed diagrams are shown in Figs. (6 and 7), respectively.

![Fig. (5). The curve of the nonlinear integral sliding mode surface s. (A higher resolution / colour version of this figure is available in the electronic copy of the article).](image)

![Fig. (6). The curve of the nonlinear integral sliding mode surface s when motor drives. (A higher resolution / colour version of this figure is available in the electronic copy of the article).](image)

![Fig. (7). The curve of the sliding mode surface s value when the motor suddenly increases the load. (A higher resolution / colour version of this figure is available in the electronic copy of the article).](image)

It can be seen from the figure that the primary of the permanent magnet linear synchronous motor under the nonlinear integral sliding mode control based on the gain-time-varying reaching law can reach the specified one faster than the primary of the permanent magnet synchronous linear motor under the traditional sliding mode control, besides, the former has a fast primary response speed and better dynamic performance. After the sudden load increase at 0.15s, the primary speed change of the permanent magnet linear synchronous motor under nonlinear integral sliding mode control based on gain time-varying reaching law is reduced by about 3/4 times compared with the traditional sliding mode, which means it has better immunity.

It can be seen from Fig. (5) that the value of the sliding surface s is about 33 when the motor is started under the traditional sliding mode control, while the variation of the
0.25 in Fig. (6). It means that when the motor is started, the traditional sliding mode control is not on the sliding surface but moves to the sliding surface after a period of time. The controller designed in the paper makes the system on the sliding surface at the beginning and has better robustness. When suddenly increasing the load of the permanent magnet linear synchronous motor, the value of s of the sliding surface designed in this paper is about one-thousandth of the value of the traditional sliding surface in Fig. 7. It is better to show that the controller designed in the paper is far superior to the traditional sliding mode control in the performance of global robustness.

Aiming at the problem that the traditional sliding mode load observer will cause the fluctuation of the load observation value, the paper adopts the symbolic tangent function mentioned above instead of the symbolic function to solve the fluctuation problem caused by the high-frequency switching of the system. The comparison of the simulation results is shown in Fig. (8). It can be seen from the figure that the fluctuation of the load observed by the load observer using the symbolic tangent function is much smaller than that caused by the traditional sliding mode observation.

CONCLUSION

In this paper, another type of nonlinear curve with "small error reduction, large error saturation" is introduced, and the gain rate time-varying reaching law is introduced to ensure that the speed controller with nonlinear integral sliding mode has excellent global robustness. As for the problem that the observation value of the load sliding mode observer has large fluctuations, this paper proposes a method of replacing the traditional symbol function by the symbol sine function, which solves the problem of large fluctuation of observation value. This paper demonstrates the stability of the speed controller and load observer and compares it with traditional sliding mode control. Finally, the nonlinear integral sliding mode controller based on gain time-varying reaching law is shown to have good global robustness and dynamic performance in permanent magnet linear synchronous motor control.

LIST OF ABBREVIATION

PMLSM = Permanent Magnet Linear Synchronous Motor

REFERENCES


