Optimal Placement of MEMS Sensors for Damage Detection in Composite Plates

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Abstract: Objective: Inertial Micro Electro-Mechanical Systems (MEMS) have recently been proposed as sensing components of surface-mounted structural health monitoring systems aimed to detect damage in flexible plates, subjected to either quasi-static or dynamic loadings.

Method: In this paper, we discuss a topology optimization approach to smartly deploy an array of MEMS accelerometers over thin flexible plates, partially allowing for scattering in their placement.

Result: We assume damage (i.e., a reduction of the local bending stiffness of the plate) to be located anywhere, and we show how boundary conditions, symmetry conditions and the stacking sequence in case of a composite laminate can affect the optimal sensor placement.

Keywords: Structural health monitoring, Micro-Electro-Mechanical Systems (MEMS), thin plates, composites, damage detection.

1. INTRODUCTION

When subjected to extreme loadings, laminated composites can dissipate energy by interlaminar cracking and/or by intralaminar damage [1-8]. The first dissipation mechanism consists of local failures by cracking in the resin-enriched regions between adjacent laminae; the second dissipation mechanism is instead linked to local rupture phenomena inside the phases constituting a single composite lamina, or along the interfaces among them. These local damage mechanisms do not necessarily lead to a failure of the whole composite structure, but typically reduce its original load-carrying capacity. It is therefore of primary importance to develop effective Structural Health Monitoring (SHM) systems for composite structures able to detect (and possibly locate) nested damages, and next foresee their structural lifetime or envisage maintenance procedures [9, 10].

Because of the composite microstructure and layup, it is usually difficult to locate damaged zones through visual inspection; for instance, interlaminar cracks might not be visible from the outside, since they get masked by undamaged outer laminae. Several embedded (i.e., buried along interlaminar surfaces) SHM systems have been recently proposed, based e.g. on optical fiber grids [11, 12] or piezoelectric sensors [13]. While these sensors can be very accurate and provide a detailed description of local strain and/or damage fields inside the composite, they actually affect the same measured fields because of their presence. In fact, the diameter of glass fibers [12] or the thickness of piezoelectric transducers [14] typically exceeds the width of the resin-enriched zones, within which they are embedded. To measure inner local fields, the SHM system thus modifies them; this sort of “uncertainty principle” can strongly reduce the applicability of embedded SHM techniques for microstructured materials like composites. Further to this, the presence of embedded transducers also affects the density and distribution of micro-defects, partially induced during the production process by the distortion of the microstructure around the sensors. In [14], it was shown that surface-mounting piezoelectric sensors can strongly enhance the durability of monitored composite plates: a lower resolution in capturing the inner fields therefore, helps extend (actually, reduce less) the lifetime of the whole structure.

Moving from those outcomes, in [15-17] we started assessing the capability of surface-mounted SHM systems for flexible plates based on inertial MEMS sensors. Since composites are light in weight, they do require an SHM system to be lightweight as well, otherwise their dynamic response to the applied loading results to be affected too much. The first proposal to adopt MEMS accelerometers to monitor the health of composites seems to date back to [18]; in that paper, the authors showed that MEMS can lead to “more rapid data acquisition, automation, remote diagnostics and unattended testing”, both for continuous and for on-demand
SHM. In [19], we experimentally showed that, by rigidly linking the MEMS (or the MEMS board) to the composite specimen to be tested, a real-time monitoring scheme for the length of a pre-existing and growing crack can be obtained. Results, in terms of monitored oscillations of the plate, turned out to be also in rather good agreement with theoretical predictions based on Bernoulli-Euler beam bending [20].

In simple test setups, like e.g. the double cantilever beam test [1], the optimal location of the sensors to be deployed to feel damage can be easily established without any analysis of the specimen response; in the more complex case of damaged plates subjected to non-uniform external excitations, optimal sensor deployment looks less trivial. Through the experimental campaign reported in [19, 20], we showed that MEMS accelerometers can effectively detect a variation in the orientation of a frame locally moving with the plate, and therefore a variation of the structural stiffness due to the presence of a damage; this result was obtained thanks to the capability of such sensors to also feel the gravity acceleration. Hence, a novel methodology to deploy sensors over a plate to be monitored, accounting for variations of the local mid-plane orientation induced by the inception/growth of damage, was proposed in [21] and further discussed in [15, 16]. Such methodology is designed to maximize the damage-induced variation of the plate response to a specific loading condition, as felt by an array of sensors whose deployment over the structure needs to be optimized. Since the topology of the sensing system has to be optimized, we borrow the algorithmic strategy from topology optimization.

Dealing with thin flexible structures like plates, in former studies we have focused on the capability of the method to capture possible clusters in the optimal topology of the sensor network induced by the location of damage [15], to also show that a uniformly spaced grid would lead to an unnecessary consumption of resources. Moreover, we have also reported how a hierarchy of topologies can be built by allowing the necessary consumption of resources. Hence, a novel methodology to deploy sensors over a plate to be monitored, accounting for variations of the local mid-plane orientation induced by the inception/growth of damage, was proposed in [21] and further discussed in [15, 16]. Such methodology is designed to maximize the damage-induced variation of the plate response to a specific loading condition, as felt by an array of sensors whose deployment over the structure needs to be optimized. Since the topology of the sensing system has to be optimized, we borrow the algorithmic strategy from topology optimization.

Dealing with thin flexible structures like plates, in former studies we have focused on the capability of the method to capture possible clusters in the optimal topology of the sensor network induced by the location of damage [15], to also show that a uniformly spaced grid would lead to an unnecessary consumption of resources. Moreover, we have also reported how a hierarchy of topologies can be built by allowing for the different length-scales that characterize the mechanics of composite plates, from the structural one down to the microscopic one [22]. In this work, we deal with another issue of the design of a sensor network: the sensitivity of the sensed response to the inaccuracies of sensor placement. More specifically, in order to retain the deterministic framework developed before and to also avoid computing the sensitivity of the sensed components of the structural response to the sensor placement, we resort to a computational approach largely insensitive to distortions of the grid represented by the candidate sites for each single sensor placement.

We first consider the sensors to be optimally deployed over a thin homogeneous plate, whose virgin mechanical properties have been for instance defined through an ad-hoc homogenization procedure, see e.g. [23]. To move towards the experimental data collected in [19, 20], we next account explicitly for part-through and interlaminar damage events, by assuming the bending stiffness of the plate to be locally reduced by the damage itself. To understand how the boundary conditions and the laminate layup may affect the sensor deployment, we investigate the solutions in case of a square plate either simply supported or built in along its whole boundary.

The remainder of this paper is organized as follows. In Section 2 we provide some details of the adopted optimization scheme, focusing on how an unknown damage location is managed. Section 3 provides a summary of the finite element modeling of thin plates subject to bending in the linear regime; some fundamental details of the mixed variational formulation proposed in [24], and a brief proof of the accuracy of the approach even in the case of a largely distorted mesh are given. In Section 4 results are reported in terms of optimal sensor deployment for simply supported and clamped square plates, at varying number of sensors to be deployed. Finally, in Section 5 some concluding remarks are gathered, along with suggestions to move towards an efficient SHM scheme for self-monitoring, smart composite structures.

2. OPTIMIZATION APPROACH

The optimal placement of an assigned set of sensors over the structure is here dealt with. We frame it within the class of structural optimization problems with unknown topology, see e.g. [25, 26]. It is further assumed that our discrete formulation handles results coming from finite element simulations.

Once the structure is space discretized, we compute the solution in terms of nodal displacements and rotations (due to the considered plate kinematics), as induced by the assigned external forces, for the undamaged plate and for the same plate affected by a local stiffness reduction of known position. We then define vectors of element-wise constant generalized displacement vectors $\hat{\mathbf{w}}_i$ in the undamaged state, and $\mathbf{w}_i$ in the damaged case. Sensors may be optimally placed where the difference $\Delta_i = \|\mathbf{w}_i - \hat{\mathbf{w}}_i\|$ is the number of finite elements in the mesh. An assigned number $N$ of sensors can be deployed over the structure by maximizing the overall sensitivity to damage, according to:

$$\max \sum_{i=1}^{n} x_i^p \|\mathbf{w}_i - \hat{\mathbf{w}}_i\|$$

s.t.

$$\sum_{i=1}^{n} x_i \leq N$$

$$0 \leq x_i \leq 1 \quad i = 1, ..., n$$

where $n$ is the number of finite elements in the mesh. Penalization of intermediate density values, so as to attain or at least approach pure 0-1 distributions, may be straightforwardly achieved by tuning the penalization exponent $p \geq 1$ in (1), see [21].

In case of a multiplicity of damaged zones or, conversely, in case of unknown damage location, we have to appropriately modify the objective function in Eq. (1) to take all of them into account. For each location of damage, say inside the $k$-th element, we compute a set of structural response variations $\Delta_{ki} = \|\mathbf{w}_{ki} - \hat{\mathbf{w}}_i\|$, $\mathbf{w}_{ki}$ being the damage case dependent solution. The resulting optimization setting then reads:
where is it assumed that is in order to allow for any possible damage location. This scheme has been shown to maximize the sensitivity of the SHM scheme to the amplitude of the measured structural response, and therefore predicts a placement of sensors where the maximum variation in the structural response is read, independently of the source.

Such approach has been targeted as dangerous from the monitoring viewpoint, see [21]; in fact, it would be safer to drive sensor locations towards regions enhancing the sensitivity of the SHM scheme to the whole structural response variation, which can be well represented by an average measure of its amplitude. To accordingly balance all the possible damage sources, the objective function is reformulated as:

\[
\frac{\max \sum_{k=1}^{n} \sum_{i=1}^{n} \frac{1}{x_i} x_i^2 \| \mathbf{w}_{ki} - \mathbf{\hat{w}}_i \|}{\sum_{i=1}^{n} x_i}
\]

\[
\text{s.t.}
\]

\[
0 \leq x_i \leq N
\]

Because of the added term in the objective function, this latter formulation is here termed the non-dimensional one.

Algorithmically, the optimal solution to problems (2) and (3) is obtained through the Method of Moving Asymptotes (MMA) [27]. This method efficiently exploits convexity and separability of the optimization problem, see e.g. [28], adopting a convex linearization of the objective functions that may be regarded as a first order Taylor expansion of them. Additional details can be found in [21].

3. FINITE ELEMENT MODELING

In the forthcoming investigation of Section 4, we shall consider a thin square plate, featuring thickness and in-plane (side) length such that . To compute displacements and rotations gathered in the vectors and (see Section 2), we therefore need a finite element formulation able to provide accurate results for this thin plate limit case.

We introduce a Cartesian (orthonormal) coordinate system , the plane being coincident with the plate mid-surface. According to the first order shear deformation theory [29, 30], under the assigned loadings the displacement field is (see [24] for further details about the notation adopted):

\[
\begin{align*}
\mathbf{u}_x &= z \partial_y (x, y) \\
\mathbf{u}_y &= -z \partial_x (x, y) \\
\mathbf{u}_z &= \mathbf{u}_z (x, y)
\end{align*}
\]

\((4)\)

In (4), \(\mathbf{\theta} = \{ \partial_x, \partial_y \}^T\) is a vector gathering the rotations of the normal to the mid-plane about the in-plane axes. Even though we are dealing with thin plates, Eqs. (4) allow to consider the effects of transverse shear deformation [29]. Curvatures and shear strain components therefore read:

\[
\mathbf{\chi} = \begin{bmatrix} \partial_y \mathbf{u}_x - \partial_x \mathbf{u}_y \\
\partial_y \mathbf{u}_y - \partial_x \mathbf{u}_x 
\end{bmatrix}
\]

\((5)\)

or, in the matrix form:

\[
\begin{bmatrix}
\mathbf{Y} = E \mathbf{\theta} + \nabla \mathbf{u}_z
\end{bmatrix}
\]

\((6)\)

where \(L\) and \(\nabla\) are differential operators, and \(E\) is a Boolean matrix.

By assuming the stress component along the out-of-plane direction to be negligible, resultants per unit length of in-plane stresses and transverse shear stresses are collected, in perfect analogy with Eq. (5), as:

\[
\begin{bmatrix}
\mathbf{M} = \begin{bmatrix} M_x \\
M_y \\
M_{xy} 
\end{bmatrix}
\mathbf{S} = \begin{bmatrix} S_x \\
S_y 
\end{bmatrix}
\end{bmatrix}
\]

\((7)\)

For homogeneous linear elastic materials, the overall constitutive relation for the plate can be written in an uncoupled fashion as:

\[
\mathbf{M} = \mathbf{D}_b \mathbf{\chi}
\]

\((8)\)

Readers are referred to [24] and [29], for a thorough description of terms appearing inside bending \(\mathbf{D}_b\) and shear \(\mathbf{D}_s\) stiffness matrices; a generalization to allow for the inhomogeneous and anisotropic properties of laminated composite plates can be found in [31].

Moving now to the finite element formulation, besides the external load terms the following mixed functional is handled:

\[
\Pi(u_x, \mathbf{\theta}, S) = \frac{1}{2} \int_A \mathbf{\chi}^T \mathbf{D}_b \mathbf{\chi} \, dA - \frac{1}{2} \int_A S^T D_s^{-1} S \, dA + \int_A S^T (E \mathbf{\theta} + \nabla \mathbf{u}_z) \, dA
\]

\((9)\)

\[\text{where } u_x, \mathbf{\theta} \text{ and } S \text{ are independently space-interpolated according to:}\]

\[
\begin{align*}
\mathbf{u}_x &= \mathbf{\Phi}_{\mathbf{u}_x} \mathbf{u}_x^h + \mathbf{\Phi}_{\mathbf{u}_x} \mathbf{\theta}_h \\
\mathbf{\theta} &= \mathbf{\Phi}_{\mathbf{\theta}} \mathbf{\theta}^h + \mathbf{\Phi}_{\mathbf{\theta}} \mathbf{\theta}_h \\
S &= \mathbf{\Phi}_{S} S_h
\end{align*}
\]

\((10)\)

Here above, the superscript \(h\) denotes values of the degrees of freedom associated with a mesh of characteristic size \(h\). While \(u_x^h\) and \(\mathbf{\theta}_h\) are associated with the mesh nodes, \(\mathbf{\theta}_h\) and \(S_h\) are internal (i.e. elemental) degrees of freedom that can be later condensed out. The latter set of degrees of freedom, and the related shape functions gathered in \(\mathbf{\Phi}_{\mathbf{\theta}}\) and \(\mathbf{\Phi}_{S}\) are to be chosen to fulfill the conditions for mixed variational formulations to be stable and consistent [32]; see [24] for details.
Before moving to the solution of the optimization problem, to check the accuracy of the adopted formulation we consider the case of the homogeneous square plate featuring \( \frac{t}{L} = \frac{1}{1000} \) and subjected to a uniform distributed load \( q \), either simply supported or clamped along its whole boundary. The relevant analytical solution, in terms of nondimensional deflection \( \bar{u}_x \) and bending moment \( \bar{M} = \bar{M}_x = \bar{M}_y \) (due to symmetry) at the center of the plate [33] is reported in Table 1. For the simply supported and clamped plates, Figs. (1 and 2) respectively provide a convergence study of the finite element results, obtained with the FEAP code [34], at increasing mesh density (measured by the number of elements \( n_{el} \) used to discretize each side of the plate), in case of structured discretizations. It can be seen that beyond \( n_{el} = 32 \), the error in the solution never exceeds 0.3\% as for the reference deflection, and 1.3\% as for the central bending moment. Although the aim of this activity is the optimization of the sensor network topology and not the analysis of damage in composite plates, plots of the bending moments are provided to testify the capability of the adopted finite element scheme to accurately capture a possible structural health evolution due to the stress-induced growth of damage.

### Table 1. Homogeneous square plate, \( \frac{t}{L} = \frac{1}{1000} \): analytical solution, in terms of central deflection \( \bar{u}_x \) and bending moment \( \bar{M} \).

<table>
<thead>
<tr>
<th></th>
<th>( \bar{u}_x / \frac{12qlx}{Et^3} )</th>
<th>( \bar{M} / qL^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simply Supported Case</td>
<td>0.00406</td>
<td>0.0479</td>
</tr>
<tr>
<td>Clamped Case</td>
<td>0.00126</td>
<td>0.0231</td>
</tr>
</tbody>
</table>

The results collected in Figs. (1 and 2) are only marginally affected by element distortion. For \( n_{el} = 32 \) and by randomizing the position of the nodes around that of the structured mesh (featuring perfectly square elements only), Fig. (3) provides maps of out-of-plane displacement at varying values of the parameter \( \rho \), which quantifies the mesh distortion as a dimensionless amplification factor of the element size. Fig. (4) shows the effect of \( \rho \) on the central deflection \( u_x \) and on the bending moment \( M \). All the graphs demonstrate that for distortions amounting up to \( \rho = 0.6 \), for a finite element discretization distorted as shown in Fig. (3c), all the relevant outcomes of the simulation are not signifi-

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**Fig. (1).** Simply supported homogeneous square plate, \( \frac{t}{L} = \frac{1}{1000} \): effect of space discretization on the accuracy of the finite element solution, in terms of central (a) deflection \( u_x \) and (b) bending moment.

**Fig. (2).** Clamped homogeneous square plate, \( \frac{t}{L} = \frac{1}{1000} \): effect of space discretization on the accuracy of the finite element solution, in terms of central (a) deflection \( u_x \) and (b) bending moment \( M \).
Robustness of the adopted modeling scheme against uncertainties, related e.g. to real-life scattered positioning of the sensors in the SHM network, is thus proved for the case under investigation.

4. RESULTS

As already anticipated, as far as the optimal deployment of a network of inertial sensors is concerned, we consider a thin square plate featuring a thickness to side length ratio $\frac{t}{L} = \frac{1}{1000}$. The plate is either simply supported or fully built in along its four edges. We start by assuming the plate to be homogeneous. Concerning the material properties, strength and toughness values are irrelevant since damage growth is of no concern; although not specifically reported, elastic moduli are instead assumed to refer to a quasi-isotropic laminate [35]. Through this analysis, we can provide outcomes of the optimization procedure of interest for isotropic (and homogeneous) materials too.

In the present investigation, it is assumed that the size of the damaged zone, wherever it is located, covers a single finite element. As already reported in [22], this is not to be considered a limitation of the proposed approach, since a hierarchy of scale-dependent analyses can be handled to also allow for the actual micrometer size of the MEMS sensors or of their boards, see [19, 20].

In a preliminary stage of the analysis, see [16], the position of damage was considered to be known in advance. We investigated the results at varying position of the damaged zone, featuring a reduction by 50% of the local bending stiffness, and we showed that the optimal solution consists of sensors located around the same damaged zone. Only in case of high $N$ values (see Eqs. 2 and 3), i.e. of several sensors to be deployed over the plate, a sort of delocalization pattern shows up. All those deployments were obtained by assuming that rotations $\theta_x$ and $\theta_y$ can be measured by the sensors; this is practically feasible since MEMS accelerometers are designed to also sense the gravity direction, and are therefore able to detect variations of their spatial orientation even under quasi-static loadings. To restore symmetry, and therefore fulfill general symmetry conditions over the whole plate, the norm $\theta = \sqrt{\theta_x^2 + \theta_y^2}$ of the rotation vector was introduced and adopted throughout.

Fig. (3). Simply supported homogeneous square plate, $\frac{t}{L} = \frac{1}{1000}$: effect of mesh distortion on the accuracy of the finite element solution, in terms of out-of-plane displacement. (a): $\rho = 0.2$; (b): $\rho = 0.4$; (c): $\rho = 0.6$; (d): $\rho = 0.8$.

Fig. (4). Simply supported homogeneous square plate, $\frac{t}{L} = \frac{1}{1000}$: effect of mesh distortion on the accuracy of the finite element solution, in terms of (a) central deflection $u_z$ and (b) bending moment $M$.
In this work, we still assume the rotation $\theta$ as the observed variable to feed the SHM system and sense damage. As described in Section 2, we also consider non-dimensional variations of $\theta$, in order to enhance the sensitivity of the measured structural response to damage, and not the sensitivity to the largest effect of damage.

Results of the optimization procedure are reported in Figs. (5 and 6), in terms of the optimal position of $N$ sensors in the simply supported and clamped cases, respectively. In both figures, a comparison is presented between results of formulation (2) (left columns) and of formulation (3) (right columns). At increasing $N$, it is shown that (2) leads to a concentration of sensors close to the center of the plate, where the lateral deflection is maximum and the rotation of the mid-plane is minimum in the undamaged case; this obviously occurs because damage induces the maximum total variation of $\theta$ in such region. In case of a clamped plate four additional regions, symmetrically placed about the center of the plate, are also expected to contribute by enhancing the sensitivity of the SHM system to damage effects, see Fig. (5) left column. If formulation (3) is adopted in the simply supported case, sensors are placed close to the midpoints of the four edges of the plate, see Fig. (5) right column. This latter result is corroborated by what shown for the clamped case (Fig. 6 right column), with sensors that need to be moved toward the center of the plate, away from the clamped boundaries along which $\theta$ is always zero, independently of damage location. These results have been provided as projected onto the structured grid shown in the pictures, but within the current deterministic frame, they can also be obtained by adopting a mildly distorted mesh to mimic possible uncertainties or unexpected errors in practical sensor location. Additional information, related e.g. to the propagation of errors from sensor location to damage detection or to the optimal network topology, can be appropriately dealt with only within a stochastic framework like e.g. that proposed in [36]. Further results relevant to other plate geometries or to different combinations of boundary conditions along the edges of the plate, do not add specific details concerning the capability of the proposed method to optimize the deployment of the sensors; accordingly, they are not discussed here for the sake of brevity.

Moving now to the composite laminate case, we keep valid all the settings related to geometry, loading conditions and damage extension, with the plate always clamped along the whole boundary. As for the stacking sequence, we consider a $[90/0]_{2s}$ and a $[\pm45]_{2s}$ symmetric configurations. At the lamina level, in a local $x_1-x_2-x_3$ reference frame with axis $x_1$ aligned with the fiber direction, axes $x_2$ and $x_3$ orthogonal to it with $x_3$ also parallel to the global out-of-plane
direction $z$, the relevant elastic moduli read: $E_{11} = 161$ GPa, $E_{22} = 11.4$ GPa, $G_{12} = G_{13} = 5.17$ GPa, $G_{23} = 3.98$ GPa, $\nu_{12} = 0.32$.

For the $[90/0]_{2\times}$ configuration, Figs. (7 and 8) gather the optimal sensor locations obtained with the dimensionless formulation of Eq. (3) at an increasing value of $\bar{N}$, by assuming that a part-through damage or a delamination along the mid-plane has to be respectively sensed. In view of the results shown to testify the sensitivity of the modelled response to the space discretization, a structured mesh less refined than the one adopted for the homogeneous case is here handled; as discussed above, results can indeed be finely tuned within the

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**Fig. (6).** Clamped homogeneous square plate, $\frac{c}{L} = \frac{1}{1000}$: optimal deployment of $\bar{N}$ sensors to monitor variations of $\vartheta$ (left column) or non-dimensional variations of $\vartheta$ (right column). (a)-(b) $\bar{N} = 4$; (c)-(d) $\bar{N} = 16$; (e)-(f) $\bar{N} = 64$.

**Fig. (7).** Clamped composite square plate, $[90/0]_{2\times}$ configuration: optimal deployment of sensors to monitor non-dimensional variations of $\vartheta$ due to a part-through damage. From left to right and from top to bottom, solutions are displayed for $\bar{N} = 1$, $\bar{N} = 2$, $\bar{N} = 4$, $\bar{N} = 8$, $\bar{N} = 10$, $\bar{N} = 12$, $\bar{N} = 14$, $\bar{N} = 16$. 
multi-scale strategy proposed in [22]. It is shown that sensors tend to be deployed in a central portion of the plate, almost independently of the damage type envisaged and still in compliance with the symmetry conditions, as allowed by the rather coarse space discretization adopted.

Fig. (8). Clamped composite square plate, [90/0]_{24} configuration: optimal deployment of sensors to monitor non-dimensional variations of $\vartheta$ due to a delamination along the mid-plane. From left to right and from top to bottom, solutions are displayed for $\bar{N} = 1$, $\bar{N} = 2$, $\bar{N} = 4$, $\bar{N} = 8$, $\bar{N} = 10$, $\bar{N} = 12$, $\bar{N} = 14$, $\bar{N} = 16$.

Fig. (9). Clamped composite square plate, [±45]_{24} configuration: optimal deployment of sensors to monitor non-dimensional variations of $\vartheta$ due to a delamination along the mid-plane. Scaled objective function and, from left to right and from top to bottom, solutions for $\bar{N} = 1$, $\bar{N} = 2$, $\bar{N} = 4$, $\bar{N} = 8$, $\bar{N} = 10$, $\bar{N} = 12$, $\bar{N} = 14$, $\bar{N} = 16$.

Fig. (9) collects instead the results of the optimization procedure for the [±45]_{24} configuration, in terms of the scaled objective function to be maximized by formulation (3) and of the corresponding optimal sensor deployment to detect a mid-plane delamination, at an increasing $\bar{N}$ value. Due to the composite layup, symmetry in the solution is preserved about the axes $x_1$ and $x_2$ of the laminae, but gets rotated by 45° in the mid-plane of the plate. As before, the placement of the sensors is characterized by a sort of clustering effect where the objective function provides the highest values, and so a larger sensitivity of measurements to damage.
To finally summarize the output of the optimization procedure, it clearly appears that sensors are never homogeneously deployed all over the plate. Hence, evenly spaced transducers would not be the optimal solution to sense a damage of unknown position, possibly nucleated and grown due to extreme (even unknown) loading conditions.

CONCLUSION

To develop a well performing SHM system for composite structures, we claim that it has to be light in weight, otherwise, the structural response to the external loads would be affected too much by the presence of the sensors. Former experimental investigations showed that embedded sensors can have a detrimental effect on the load-bearing capacity and on the durability of the structure, because of defects caused by sensor embedding; hence, we have proposed to adopt an approach based on surface-mounted MEMS sensors (actually accelerometers). Since very cheap commercial-off-the-shelf MEMS are currently available on the market [36], we formerly focused our investigation on their capacity to match accuracy requirements for a rather standard monitoring system.

In this work, we have provided a numerical study to move towards an efficient SHM procedure for composite laminates exposed to damage growth due to extreme loading conditions. In [19, 20] we experimentally assessed the reliability and robustness of MEMS accelerometers to detect damage or delamination growth under controlled external actions; here, we have instead assumed the health of the structural component to be unknown, and proposed a topology optimization of sensor deployment to maximize the sensitivity to damage of the monitoring scheme. The obtained results are independent of damage location and are also somehow robust against possible uncertainties related to real-life placement of the sensors. Outcomes of the proposed procedure have shown that, no matter what the boundary conditions are, a homogeneously deployed array of sensors would not be the optimal provision to effectively locate damage (or delamination, thought as a local reduction of the bending stiffness) in thin plates.

Results have been reported for flexible plates only but, as the procedure is based upon a very general formulation to maximize the sensitivity to damage of the measurements collected through the SHM system, the same approach can be used also for structures characterized by another kinematics. Since the whole procedure is centered around a comparison between damage-free and damage-affected numerical solutions, the solutions themselves need to be accurate to obtain effective sensor network geometries. For the specific cases here analyzed, in accordance with Saint-Venant’s principle it may happen that the plate kinematics does not prove appropriate to describe fields close to stiffness jumps due to damage or thickness variation. Another limitation of the current approach can be linked to the relevant purely deterministic perspective, not able to account for measurement noise and model inaccuracies [37-39].

To assess the effects of the aforementioned possible limitations, an efficiency and accuracy investigation of the proposed methodology, together with a study to automatically set the number of sensors to be deployed (now assumed known a-priori) on the basis of their accuracy, will be presented in future investigations.

CONSENT FOR PUBLICATION

Not applicable.

CONFLICT OF INTEREST

The authors declare no conflict of interest, financial or otherwise.

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