The Amplitude Reduction of a Flexible Rotor Passing the Critical Speed by Modulating Acceleration and Stiffness Simultaneously

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Abstract: Background: When an unbalanced flexible rotor passes through its critical speed, it is very easy to result in great vibration amplitude, even the destruction of rotor and bearings. Therefore, it is significant for high-speed rotating machines to reduce the vibration amplitude of flexible rotor. Various patents have been discussed in this article.

Objective: The purpose of this study is to find out the amplitude reduction method and simulate the feasibility of flexible rotor passing through critical speed.

Methods: On the basis of the model of single disk rotor system with eccentric mass, a novel method is presented to reduce the vibration amplitude passing through critical speed by modulating the acceleration and stiffness simultaneously. Firstly, the amplitude characteristic of a flexible rotor with different acceleration and supporting stiffness was investigated. Then, the method of changing the stiffness during variable acceleration was carried out by numerical simulation based on Newmark algorithm. Furthermore, the strain energy of rotor and input power were also analyzed by using the method of simultaneous modulation of acceleration and stiffness.

Results: The simulation results revealed that the simultaneous modulation of acceleration and stiffness could reduce the vibration amplitude of rotor effectively, which was reduced by 44% and 13%, comparing with the single variable acceleration and the single variation of stiffness, respectively. Moreover, the variation tendency of total energy was similar to that of the rotor amplitude, which could be controlled at a very small level. The input power was mainly dependent on the acceleration, but had very little to do with the stiffness.

Conclusion: The method was suitable for the model of single disk rotor system, and it could also be applied to complex rotor systems, which was very useful for the security of high speed rotating machine.

Keywords: Acceleration, amplitude reduction, critical speed, flexible rotor, Newmark algorithm, stiffness.

1. INTRODUCTION

Rotor is the key part of rotating machines, such as aero-engine, electromotor, gas turbine and pump etc. As the rated speed of flexible rotor is higher than its critical speed, the rotor will cross its critical speed when it starts or stops. When the rotor passes through its critical speed, the unbalanced mass may cause great vibration amplitude, which can evenly destroy the rotor and bearings. Therefore, it is significant for high-speed rotating machines to reduce the vibration amplitude of the flexible rotor.

In industry practices, one common method to avoid the large resonance amplitude at critical speed is to use a larger acceleration when passing through the critical speed. Cao et al. [1] proposed a time-transient method for solving coupled lateral and torsional analysis of a flexible rotor-bearing system. Ishida et al. [2] investigated the transient vibration of a rotor with nonlinear spring characteristic, and found that it was more difficult to pass through the critical speed in the nonlinear system at a constant acceleration. Haehner et al. [3] and Hohl et al. [4] put forward the variable acceleration or deceleration schedule to reduce the lateral vibration amplitude, which had been verified by the numerical simulation and experiment. Some authors [5-8] proposed phase modulation method in the application of unbalanced shaft startup. Zhu et al. [9] studied the vibration reduction method and device by combined strategy. Konkola et al. [10] studied the fast acceleration reaction force cancelling motor, which is suited to precision machinery that needs to quickly accelerate. Gergely et al. [11] proposed acceleration sensing method and system for operating restraint devices. Maalouf et al. [12] proposed a new method to reduce the vibration amplitude of a flexible rotor system.
[12] proposed the method of detecting rotor anomalies during transient speed operations. Pettersson et al. [13] proposed the method for accelerating a hybrid vehicle. Zimmer et al. [14] proposed a method and a device for accelerating a gear train driving on a block. Zheng et al. [15] analyzed the transient vibration of single-thick-disk rotor crossing two orders of critical speeds. On the other hand, when the lateral displacement of the shaft was reduced by increasing acceleration, a limited torque input has also been studied [16]. Song [17] studied the vibration measurement of rotor acceleration sensor based on Labview. Yu et al. [18] studied the effects of acceleration on the transient response of magnetic suspension rotor. Strautmanis et al. [19] studied the impact of rotor elastic suspension settings on the acceleration of the automatic balancer compensating mass. Liukku et al. [20] studied the capacitive micromechanical acceleration sensor.

Another common method of amplitude reduction for a flexible rotor is frequency tuning, which is carried out by changing the stiffness of rotor support system to avoid the critical speed. In this method, the stiffness decreases rapidly when the speed of the rotor is close to the critical speed, and it increases again after exceeding the critical speed. There are several designing methods to change the stiffness of the supports. Zapomel and Ferfecki [21] studied the method of reducing lateral vibration of rotors by tuning the stiffness of rolling-element bearings. Ishida et al. [22] proposed a new vibration suppression method by utilizing the discontinuous spring characteristic. Some researchers [23-25] adopted the active control method to reduce resonance amplitude by changing the support stiffness of rotating superconducting rotors. Lu et al. [26] studied the lateral vibration in a hydroelectric generating set under various supporting conditions, in which the stiffness of tilting pad thrust bearing was changed. Chouksey [27, 28] studied the modal of rotating shaft passing through resonances by switching the shaft stiffness, which would yield small resonance deflections. Zhu et al. [29] found that the variable stiffness damping support was very effective to control vibration of the rotor, which could successfully be used in the real rotating machinery. Lopez-Medina and Silva-Navarro [30], Ortega et al. [31, 32] presented an active vibration control scheme to reduce the vibration in rotor-bearing systems, which was supported on two ball bearings, and one of the bearings can be automatically moved to change the rotor stiffness. Elfass et al. [33] proposed the method and loading module to mechanically increase the end bearing stiffness of pile/drilled shaft. Lou et al. [34] studied the mining support with supporting stiffness capable of being automatically adjusted. Jiang et al. [35] proposed the active air bearing device, which improved the dynamic stiffness characteristics. Faber et al. [36] proposed the switchable bearing bush for a motor vehicle to change the stiffness. Madge [37] studied the bearing arrangement and cage, which were suitable for gas turbine engine with high stiffness. Post [38, 39] proposed the Halbach-array configuration for levitating passive magnetic bearing. Ertas et al. [40] improved the bearing damper with external support spring systems and methods. Yan et al. [41] studied variable stiffness positioning device for railway vehicle bogie axle box. Suissa et al. [42] and Churchill et al. [43] studied a device and system using negative stiffness. Modrzejewski [44] studied the flight control device including electro rheological elastomeric bearing with variable stiffness. Teimel et al. [45] studied an electric direct-current motor with flexible rotor assembly and the method of the manufacturing. Karaki [46] studied vibrating gyroscope for car navigation systems, etc. Pallari et al. [47] studied support apparatus with adjustable stiffness. Kumar et al. [48] studied the transition duct support and proposed a method to provide a tuned level of support stiffness. Meacham et al. [49] proposed methods for controlling non-synchronous vibrations in rotating machinery using rotor support structures including anisotropic foil bearings or anisotropic bearing housings. Werner [50] analyzed the influence of electromagnetic field damping on forced vibrations of induction rotors caused by dynamic rotor eccentricity. Ferfecki et al. [51] analyzed the vibration attenuation of rotors supported by magnetorheological squeeze film dampers. Bab et al. [52] analyzed the vibration attenuation of a continuous rotor-blisk journal bearing system employing smooth nonlinear energy sinks. Eissa et al. [53] studied the nonlinear vibration control of a horizontally supported Jeffcott-rotor system. He et al. [54] studied the squeeze film damper effect on vibration of an unbalanced flexible rotor using harmonic balance method. Zadorozhnaya et al. [55] studied theoretical and experimental investigations of the rotor vibration amplitude of the turbococharger. Wang et al. [56] studied a magnetorheological fluid lubricated floating ring bearing used to control the rotor vibration. Jun et al. [57] analyzed the vibration signal of engine rotor. Wu et al. [58] studied the nonlinear dynamics near resonances of a rotor supported by active magnetic bearings system with varying stiffness. Seok et al. [59] studied equivalent stiffness and damping coefficient for ball bearing-rotor system supported on metal mesh dampers. Brito et al. [60] proposed experimental estimation of journal bearing stiffness for damage detection in large hydro generators. Zhang et al. [61] analyzed the response characteristics of a rotor-bearing system with double frequency time-varying bearing stiffness.


From the above literatures, it can be concluded that the modulation of single rotor acceleration, single support stiffness or vibration attenuation device is the usual method to reduce the lateral vibration amplitude of rotor. However,
there are few reports concerning the amplitude reduction by changing the stiffness during rotor variable acceleration. In this paper, a new method of simultaneous modulation of acceleration and support stiffness for amplitude reduction was proposed, which was verified by Newmark algorithm in MATLAB simulation. In the numerical simulation, the transient response of flexible rotor system was studied, while the vibration amplitude characteristic of the rotor was systematically analyzed under variable rotor acceleration, different supporting stiffness, simultaneous changing of acceleration and stiffness. And the simulation results revealed that the simultaneous modulation of supporting stiffness and acceleration could reduce the rotor resonance amplitude effectively, which was smaller than that of the single variable acceleration or single stiffness adjusting at critical speed.

2. MATERIALS & METHODS

The model of rotor system is a single disk rotor, which consists of a rigid eccentric disk and a shaft, as shown in Fig. (1). In the fixed coordinate system, axis $z$ is identical with the undeformed centre line of the shaft, origin $O$ lies in the middle plane of the disc, axes $x$ and $y$ are perpendicular to axis $z$. The disk mass is described as $m_d$, and the ends of the shaft are supported by bearings. The bearing is simplified to be elastic damping support, which is uncoupled in $x$ and $y$ direction.

$$T_a = \frac{1}{2} m_d \left( \ddot{x}_a \dot{y}_a + \ddot{y}_a \dot{x}_a \right) + \frac{1}{2} m_d \left( \ddot{x}_a \dot{y}_a + \ddot{y}_a \dot{x}_a \right)$$

By adding up Eqs. (3) and (4), the kinetic energy of rotor system can be given as;

$$T = \frac{1}{2} \dot{q}_s \begin{bmatrix} M_1 & M_2 \\ M_2 & M_3 \end{bmatrix} \dot{q}_s + \frac{1}{2} \dot{q}_s \begin{bmatrix} G_1 \\ G_2 \end{bmatrix} q_s$$

$$\text{where}$$

$$M_1 = \begin{bmatrix} m_d & J_d & m_s \\ J_d & m_s & m_s \\ m_s & m_s & m_s \end{bmatrix}$$

$$G_1 = \begin{bmatrix} J_p & 0 \\ 0 & 0 \end{bmatrix}$$

Assuming the rotor system is axisymmetric, the elastic potential energy of shaft may be written as Eq. (8):

$$V_d = \frac{1}{2} \dot{q}_s \begin{bmatrix} K_1 & \Phi K_2 \\ \Phi K_2 & K_2 \end{bmatrix} \frac{1}{2} \dot{q}_s$$

where, $K_1$ is the stiffness matrix of the shaft, $K_2$ is the elastic stiffness matrix under bend deformation, $\Phi$ is the biasing matrix due to the degree of freedom of shaft.

$$K_1 = \begin{bmatrix} K_{xx} & -K_{xy} \\ -K_{xy} & K_{yy} \end{bmatrix}$$

$$K_2 = \frac{E}{l^2} \begin{bmatrix} 12 & -6I \\ -6I & 4I \end{bmatrix}$$

And the potential energy of two end nodes of the shaft is;

$$V_a = \frac{1}{2} \begin{bmatrix} k_a & k_b \\ k_b & k_a \end{bmatrix} \begin{bmatrix} x_a^2 + y_a^2 \\ x_b^2 + y_b^2 \end{bmatrix}$$

The stiffness of bearings is expressed as:

$$K_s = \begin{bmatrix} 0 & k_a \\ k_a & k_b \end{bmatrix}$$

And the stiffness of rotor system is given as:

$$K_s = K_1 + K_2$$

Adding up Eqs. (8) and (12), the potential energy of rotor system may be written as:

$$V = V_d + V_a$$
The disk is shown in Fig. (2). The coordinate system $Tx'y'$ is selected, and its origin $T$ is situated in the disc centre of the mass (gravity). It moves together with the disc, but its coordinate axes $x'$, $y'$ remain parallel with axes $x$, $y$ of the fixed reference frame $(Tx'y')$ performs only a translation motion.

Fig. (2). Reference frames of the disk.

The rotor starts from the rest at $\varphi = 0$ and $t = 0$. Then, the centrifugal and torsional force of disk can be written as:

$$F_c = m_e\omega^2\varphi$$

(16)

$$F_t = m_e\omega\dot{\varphi}$$

(17)

where $F_c$ is the centrifugal force, $F_t$ is the torsional force, $m_e$ is the eccentric distance of the disk, $\omega$ is the angular velocity, and $\dot{\varphi}$ is the angular acceleration. Then the generalized force of rotor system in the $x$ and $y$ directions can be derived by:

$$Q_j = \{F_x, 0, 0, F_y, 0, 0, 0\}^T$$

(18)

$$F_x = F_c \cos \varphi + F_t \sin \varphi$$

(19)

$$F_y = F_c \sin \varphi - F_t \cos \varphi$$

(20)

And the damping of bearings is given as:

$$C_b = \begin{bmatrix} 0 & 0 \\ 0 & C_d \\ C_g & 0 \end{bmatrix}$$

(21)

The Lagrange equation of motion may be expressed as:

$$\frac{d}{dt} \left( \frac{\partial T}{\partial \dot{q}_j} \right) - \frac{\partial T}{\partial q_j} + \frac{\partial V}{\partial \dot{q}_j} = Q_j$$

(22)

Substituting Eqs. (5), (15) and (18) into Eq. (22), and the motion equation of rotor system is obtained.

$$\begin{bmatrix} M_1 & M_1 \\ M_1 & M_1 \end{bmatrix} \begin{bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \end{bmatrix} + \Omega \begin{bmatrix} C_a & G_1 \\ -G_1 & C_a \end{bmatrix} \begin{bmatrix} \dot{u}_1 \\ \dot{u}_2 \end{bmatrix} = \begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix}$$

(23)

$$\begin{bmatrix} K_e & 0 \\ 0 & K_e \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix}$$

(24)

Then the motion equation of bearing-rotor system can be obtained as follows:

$$\begin{bmatrix} M \ddot{u} \end{bmatrix} + \begin{bmatrix} C \end{bmatrix} \dot{u} + \begin{bmatrix} K \end{bmatrix} u = \begin{bmatrix} \dot{Q} \end{bmatrix}$$

(25)

By solving Eq. (24) based on Newmark algorithm, the displacements $x(t)$ and $y(t)$ of rotor in $x$ and $y$ direction can be obtained, which change over time. Hence, the vibration amplitude of rotor in radius direction can be given by:

$$r(t) = \sqrt{x(t)^2 + y(t)^2}$$

(26)

The materials and parameters of the rotor system are shown in Table 1, which are used to solve Eqs. (25, 26 & 27) by Newmark algorithm. Following this, the vibration amplitude curve of the rotor with angular velocity can be obtained.

Table 1. Parameters of Rotor System.

<table>
<thead>
<tr>
<th>Materials of Shaft and Disk</th>
<th>40Cr</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diameter of disk</td>
<td>7.62cm</td>
</tr>
<tr>
<td>Thickness of disk</td>
<td>2.45cm</td>
</tr>
<tr>
<td>$C_a = C_b$</td>
<td>7.087N.s/m</td>
</tr>
<tr>
<td>$k_a = k_b$</td>
<td>40000N/m</td>
</tr>
<tr>
<td>Unbalance</td>
<td>0.0305 kg.mm</td>
</tr>
<tr>
<td>$m_d$</td>
<td>1.0807kg</td>
</tr>
</tbody>
</table>

3. NUMERICAL SIMULATION AND ANALYSIS

3.1 Amplitude Curve of Variable Acceleration

The angular acceleration of rotor is 10rad/s$^2$, 100rad/s$^2$, 500rad/s$^2$, 1000rad/s$^2$ respectively, and the other parameters of the rotor system are the same as that shown in Table 1. After the calculation of Eqs. (25-27), the amplitude curve of the rotor is obtained, which is shown in Fig. (3). It can be seen that the maximum amplitude always occurs at the critical speed, and the amplitude decreases as the acceleration increases. The maximum amplitude is about 43μm when the rotor angular acceleration is 1000rad/s$^2$, while it is about 150μm with the acceleration of 10rad/s$^2$. Therefore, a big constant acceleration is very effective to decrease the rotor amplitude at critical speed.
In addition, it is also obvious that the critical speed of rotor increases as the acceleration increases. The critical speed is about 900rpm when the angular acceleration of rotor is 10rad/s^2, while it is about 1300rpm with the acceleration of 1000rad/s^2. So the variable acceleration method can be used to avoid the critical speed, and the essence of which is to keep away from the maximum amplitude, as shown in Fig. (4).

![Amplitude curve of rotor with different acceleration](image1)

**Fig. (3).** Amplitude curve of rotor with different acceleration.

![Vibration amplitude change with variable acceleration method](image2)

**Fig. (4).** Vibration amplitude change with variable acceleration method.

In Fig. (4), point A is the intersection of two curves of vibration amplitude, one of which corresponds to the acceleration of 1000rad/s^2, the other corresponds to the acceleration of 10rad/s^2. If the variable acceleration method is adopted during run-up process, with acceleration being 1000rad/s^2 before point A, and 10rad/s^2 after point A, then the value of point A becomes the maximum amplitude. It is about 27μm in point A, which is 37% smaller than the maximum amplitude under the constant acceleration of 1000rad/s^2 (about 43μm). Comparing to the constant big acceleration, the maximum amplitude can be reduced more effectively by using the variable acceleration method.

### 3.2. The Influence of Stiffness on the Vibration Amplitude of Rotor

The parameters are kept constant as that in Table 1, while the stiffness is 90000N/m instead of 40000N/m. Then the amplitude curve with different acceleration is drawn, as shown in Fig. (5). It can be seen that the critical speed becomes greater, while the vibration amplitude of rotor becomes smaller when the supporting stiffness of rotor becomes larger. In Fig. (5), the vibration amplitude of rotor is about 47μm at the acceleration of 1000rad/s^2, which is almost equal to the amplitude value (43μm) with stiffness of 40000N/m. In other words, the support stiffness has a little effect on the maximum amplitude under big acceleration.

![The rotor amplitude curve with stiffness of 90000N/m](image3)

**Fig. (5).** The rotor amplitude curve with stiffness of 90000N/m.

When the acceleration is 10rad/s^2, the critical speed corresponding to maximum amplitude is about 900rpm with the stiffness of 40000N/m in Fig. (3), while it is about 1300rpm as the stiffness is 90000N/m in Fig. (5). Therefore, it can be concluded that changing support stiffness is effective for reducing rotor amplitude at critical speed, and the resonance amplitude can be avoided by the change of stiffness, which is also another effective method for the amplitude reduction.

Similarly, the variable acceleration schedule is adopted to reduce vibration amplitude under different stiffness, as shown in Fig. (6).

![The rotor amplitude curve with variable acceleration schedule](image4)

**Fig. (6).** The rotor amplitude curve with variable acceleration schedule.

The maximum amplitude is about 34μm (point B1) when stiffness is 90000N/m, while it is about 33μm (point B2) when stiffness is 120000N/m. The value of point B1 almost equals to that of point B2, which means the maximum amplitude almost is unchangeable in variable acceleration method, when the stiffness increases to a certain value. Contrasting Fig. (4) with Fig. (6), it can be concluded that the essence of the variable acceleration schedule is to avoid the resonance amplitude by changing acceleration, and the resonance amplitude of rotor decreases slowly as the support stiffness decreases.

From the above analysis, it can also be seen that the vibration amplitude is small at a big acceleration (1000rad/s^2), while the vibration zone becomes wide. Thus, when the
Amplitude Reduction of Rotor by Modulating Acceleration and Stiffness

Fig. (6). Amplitude change of rotor with variable acceleration under different stiffness.

method of changing stiffness is adopted to reduce vibration amplitude, the acceleration should be kept at a small value. The acceleration is 10rad/s², while the stiffness is 40000N/m, 90000N/m, 120000N/m, respectively. Then the vibration amplitude curves of the rotor can be obtained as shown in Fig. (7). When the stiffness changes from 90000N/m to 40000N/m at point C1, 120000N/m to 40000N/m at point C2, the maximum amplitude is about 18µm (point C1), 15µm (point C2), respectively. If the change range of stiffness becomes larger, then the vibration amplitude of rotor could be much smaller by changing stiffness.

3.3. Amplitude Curve under Variable Stiffness and Acceleration

From the above analysis results, several conclusions can be obtained. First of all, the essence of variable acceleration schedule is to avoid the resonance amplitude by changing acceleration, which is more effective than constant big acceleration. Secondly, the variation in stiffness can significantly change the critical speed and avoid the resonance amplitude. Thirdly, the range of change in the acceleration or stiffness is much larger, therefore, the vibration amplitude can be reduced more obviously. As a result, if the distance between the two critical speeds can be increased by changing the stiffness during variable acceleration, the vibration amplitude will become much smaller than single acceleration or stiffness modulation.

Then, the stiffness and acceleration are selected at 40000N/m and 10rad/s², 90000N/m and 1000rad/s², 120000N/m and 1000rad/s², respectively. The amplitude curves under different stiffness and acceleration can be obtained, as shown in Fig. (8).

Fig. (7). Rotor amplitude under different stiffness with acceleration 10rad/s².

Fig. (8). The rotor amplitude curve under different stiffness and acceleration.

When the variation schedule of acceleration and stiffness varies from 1000rad/s² and 90000N/m to 10rad/s² and 40000N/m, the maximum amplitude is about 15µm at point D1, which is about 44% smaller than the amplitude (27µm) of variable acceleration method shown in Fig. (4). It is about 13µm at point D2, when the variation is from 1000rad/s² and 120000N/m to 10rad/s² and 40000N/m, which is about 52% smaller than that of the variable acceleration schedule shown.
In order to observe the reduction effect of amplitude between the single stiffness change and simultaneous modulation, the amplitude curves of rotor with different methods are drawn in Fig. (9).

![Amplitude Curves](image)

**Fig. (9).** Maximum amplitude point with different method.

In Fig. (9), points C1 and C2 are the maximum amplitude of stiffness change method, while points D1 and D2 are the maximum amplitude of simultaneous modulation. It can be seen that the value of point D1 is 17% smaller than point C1, while the value of point D2 is 13% smaller than point C1. It means that the vibration amplitude of rotor by simultaneous modulation of variable stiffness and acceleration is always smaller than single stiffness changing or variable acceleration method.

### 4. POWER AND STRAIN ENERGY CONSIDERATIONS

When the vibration amplitude of rotor is reduced by variable acceleration and stiffness schedule, it is hoped that the strain energy of rotor can be removed or absorbed quickly for small amplitude. Meanwhile, it is also hoped that the input power is to be small for low power consumption.

The total energy of rotor system is the sum of kinetic energy and elastic potential energy, which can be given as:

\[ E_t = T + V \] \hspace{1cm} (28)

According to Eq. (28), the total energy of the rotor with different stiffness and acceleration can be obtained, as shown in Fig. (10). It can be seen that the changing curve of total energy is similar to the curve of rotor amplitude, and the maximum of total energy always corresponds to the critical speed of the rotor, which is the same with the critical speed of the maximum amplitude. The simultaneous modulation of stiffness and acceleration can get lower total energy than single variable acceleration or single stiffness variation, indicating that the simultaneous modulation is very effective for reducing lateral vibration amplitude of the rotor.

In order to analyze the input power to the shaft, the external torque is an important parameter to be considered, which determines the value of acceleration and input power. For an accelerating rotor, the external torque equals the sum of acceleration moment and resistance moment, and it can be written as:

\[ M_{ex} = M_a + M_0 \] \hspace{1cm} (29)

where \( M_{ex} \) is the external torque, \( M_a \) is the acceleration moment, \( M_0 \) is the resistance moment, and the initial value of which is 20N.cm.

In addition, the acceleration moment is;

\[ M_a = J_p \dot{\phi} \] \hspace{1cm} (30)

where \( J_p \) is the polar moment of inertia of the rotor.

Combing Eqs. (29) and (30), the external input power to the shaft, \( P_{in} \), can be given by;

\[ P_{in} = M_{ex} \omega = (J_p \cdot \dot{\phi} + M_0) \omega \] \hspace{1cm} (31)

where \( \omega \) is the instantaneous angular velocity, which is equal to \( \dot{\phi} \).

According to Eq. (31), the external input power curves under different acceleration and stiffness can be plotted, as shown in Figs. (11 & 12).

![Power Curves](image)

**Fig. (11).** The power change with constant acceleration.
From Fig. (11), it can be seen that the input power increases with the increasing spin speed, and the acceleration is high, then the input power also increases more quickly, and the max input power is about 410w when the acceleration is 1000rad/s². In other words, the stiffness has a little effect on the input power, which mainly depends on the acceleration.

In Fig. (12a), the maximum input power is about 140w, which corresponds to the simultaneous modulation of acceleration method shown in Fig. (8), while it is about 116w in Fig. (12b), which corresponds to variable acceleration method shown in Fig. (4). From Fig. (12), it can be concluded that the amplitude reduction of rotor by changing acceleration increases the consumption of input power.

5. RESULTS & DISCUSSION

The simulation results revealed that the simultaneous modulation of acceleration and stiffness could reduce the vibration amplitude of rotor effectively, which was reduced by 44% and 13%, compared with the single variable acceleration and single variation of stiffness, respectively. Moreover, the variation tendency of total energy was similar to that of the rotor amplitude, which could be controlled at a very small level. And the input power was mainly dependent on acceleration, but had a very little impact on the stiffness.

Furthermore, the adjustment of supporting stiffness is very important for the feasibility of the proposed method, and there are some methods for adjusting the supporting stiffness, such as magnetic bearings, superconducting magnetic bearing, air bearing, rolling bearings coupled with foundation plate by springs of controllable stiffness, etc. The authors proposed to design a magnetic device for levitating the rolling bearings, combining the principle of magnetic bearing, and the device is still in the process of study. In the future when the magnetic levitating device will be made, this question will be analyzed and experimented in detail.

CONCLUSION

In this paper, a novel method has been proposed, which is used to reduce the vibration amplitude at critical speed by modulating the acceleration and stiffness simultaneously, and the numerical simulation on the transient response of rotor was carried out based on Newmark algorithm. The conclusions obtained show that the method was useful for amplitude reduction of single disk rotor, and it could also be applied to complex rotor systems. The conclusions are very sound for the security of high speed rotating machines.

CURRENT & FUTURE DEVELOPMENTS

Current and future developments are as follows:

1. The vibration amplitude of rotor decreases with the increase of rotor acceleration, while the input power increases dramatically with the increasing acceleration. So reducing vibration amplitude by big acceleration may cause great power consumption, which is not ideal for amplitude reduction. Therefore, the variable acceleration method is a good choice for small amplitude and input power.

2. The support stiffness is used to alter the critical speed of the rotor, and adjusting supporting stiffness and acceleration simultaneously can reduce the vibration amplitude of rotor by about 44% than single variable acceleration, and about 13% than changing stiffness.

3. Simultaneous modulation of acceleration and stiffness is a novel method for the vibration amplitude reduction of the rotor. This method corresponds to small amplitude, strain energy, and relatively small input power.

4. Ongoing and future work will focus on the design of changing and monitoring device of supporting stiffness. Furthermore, the stability of rotor system will be studied and tested when the acceleration and supporting stiffness are modulated, and rotor system driven by prime mover should be analyzed as a whole.

CONSENT FOR PUBLICATION

The authors provided consent to publish the manuscript.

CONFLICT OF INTEREST

The authors declare no conflict of interest, financial or otherwise.
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Hongchang Ding and Maoyuan Li contributed to the new method of amplitude reduction, Huibin Fu and Maoshun Li designed the simulations, and Hongchang Ding wrote the paper.

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